Quiz 9 — MA261 — July 28, 2017

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1. (10 points) Find a parametric representation of the part of the surface $x^2 + y^2 = 4z^2$ that lies above the xy-plane.

Switch to cylindrical coordinates. Then the equation becomes $r^2 = 4z^2$. Since we are looking above the xy-plane, this means $r = 2z$, or $z = r/2$. r and θ are the free variables, and the parametric representation is $\langle r\cos\theta, r\sin\theta, r/2\rangle$ where $0 \le \theta < 2\pi$, $r > 0$.

2. (10 points) Evaluate $\iint_S x^2yz\ dS$, where S is the part of the plane $z = 1 + 2x + 3y$ that lies above the rectangle $[0,3] \times [0,2]$.

Here $\frac{\partial z}{\partial x} = 2$ and $\frac{\partial z}{\partial y} = 3$ so $\sqrt{(\frac{\partial z}{\partial x})^2 + (\frac{\partial z}{\partial y})^2 + 1} = \sqrt{14}$.

$$\begin{aligned}
\iint_S x^2yz\ dS &= \sqrt{14}\int_0^2\int_0^3 x^2y(1+2x+3y)\ dx\ dy \\
&= \sqrt{14}\int_0^2\int_0^3 x^2y + 2x^3y + 3x^2y^2\ dx\ dy \\
&= \sqrt{14}\int_0^2 \frac{1}{3}x^3y + \frac{1}{2}x^4y + x^3y^2\Big|_0^3\ dy \\
&= \sqrt{14}\int_0^2 9y + \frac{81}{2}y + 27y^2\ dy \\
&= \sqrt{14}(\frac{9}{2}y^2 + \frac{81}{4}y^2 + 9y^3)\Big|_0^2 \\
&= \sqrt{14}(18 + 81 + 72) \\
&= 171\sqrt{14}
\end{aligned}$$