§1.3 Inverse, Exp & Logarithmic Functions

Given a function f, its <u>inverse</u> (if it exists) is the function f^{-1} such that whenever $y = f(x)$ then $x = f^{-1}(y)$.

$$\begin{tikzcd} x \arrow[r, mapsto, "f"] \arrow[rr, bend right] & y = f(x) \arrow[r, mapsto, "f^{-1}"] & x \end{tikzcd}$$

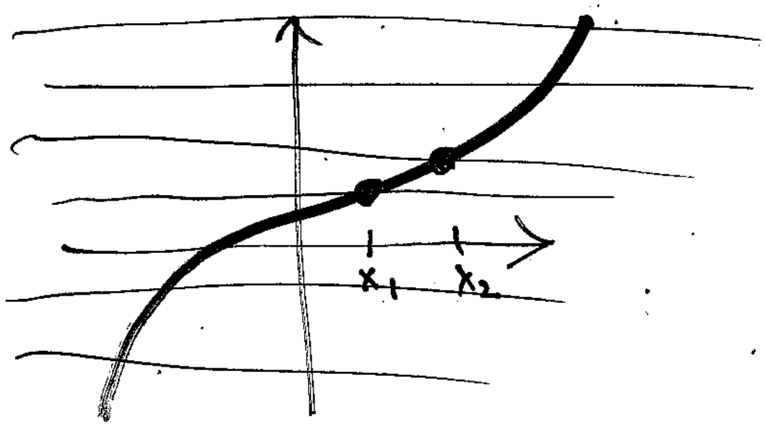
$$f^{-1}(f(x)) = x$$

Also

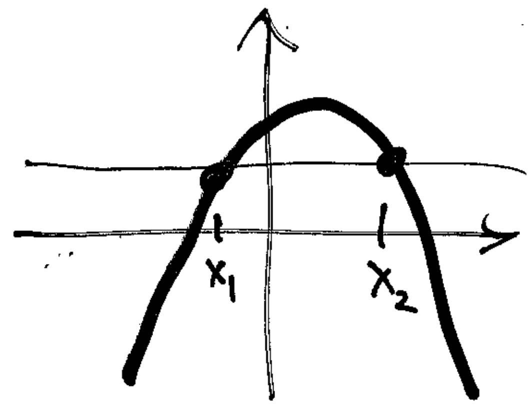
$$\begin{tikzcd} y \arrow[r, mapsto, "f^{-1}"] \arrow[rr, bend right] & x \arrow[r, mapsto, "f"] & y \end{tikzcd}$$

$$f(f^{-1}(y)) = y$$

A function f is <u>one-to-one</u> (1-1) on a domain D if $f(x_1) \neq f(x_2)$ whenever $x_1 \neq x_2$ (i.e., graph of f passes the <u>Horizontal</u> <u>Line</u> <u>Test</u>)



f is 1-1



f is not 1-1

<u>Fact</u>: If f is 1-1 on an interval I, then f has an inverse.

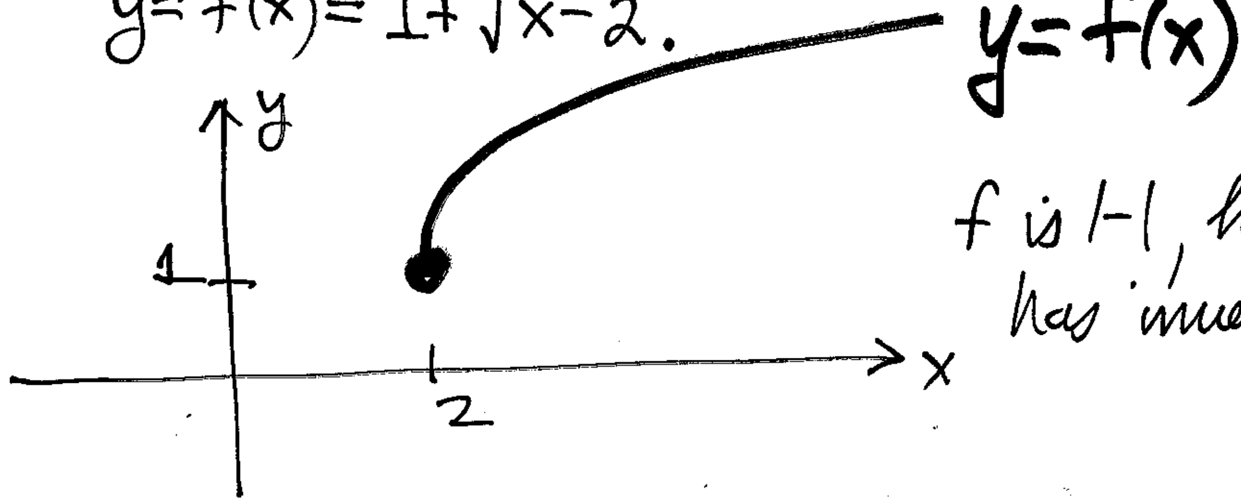
<u>Finding Inverse of $y = f(x)$</u>

① Solve for x in $y = f(x)$.

② Interchange x and y and hence $y = f^{-1}(x)$

$\boxed{\text{Ex 1}}$ Find the inverse (if possible)

$y = f(x) = 1 + \sqrt{x-2}$.



f is 1-1, hence has inverse

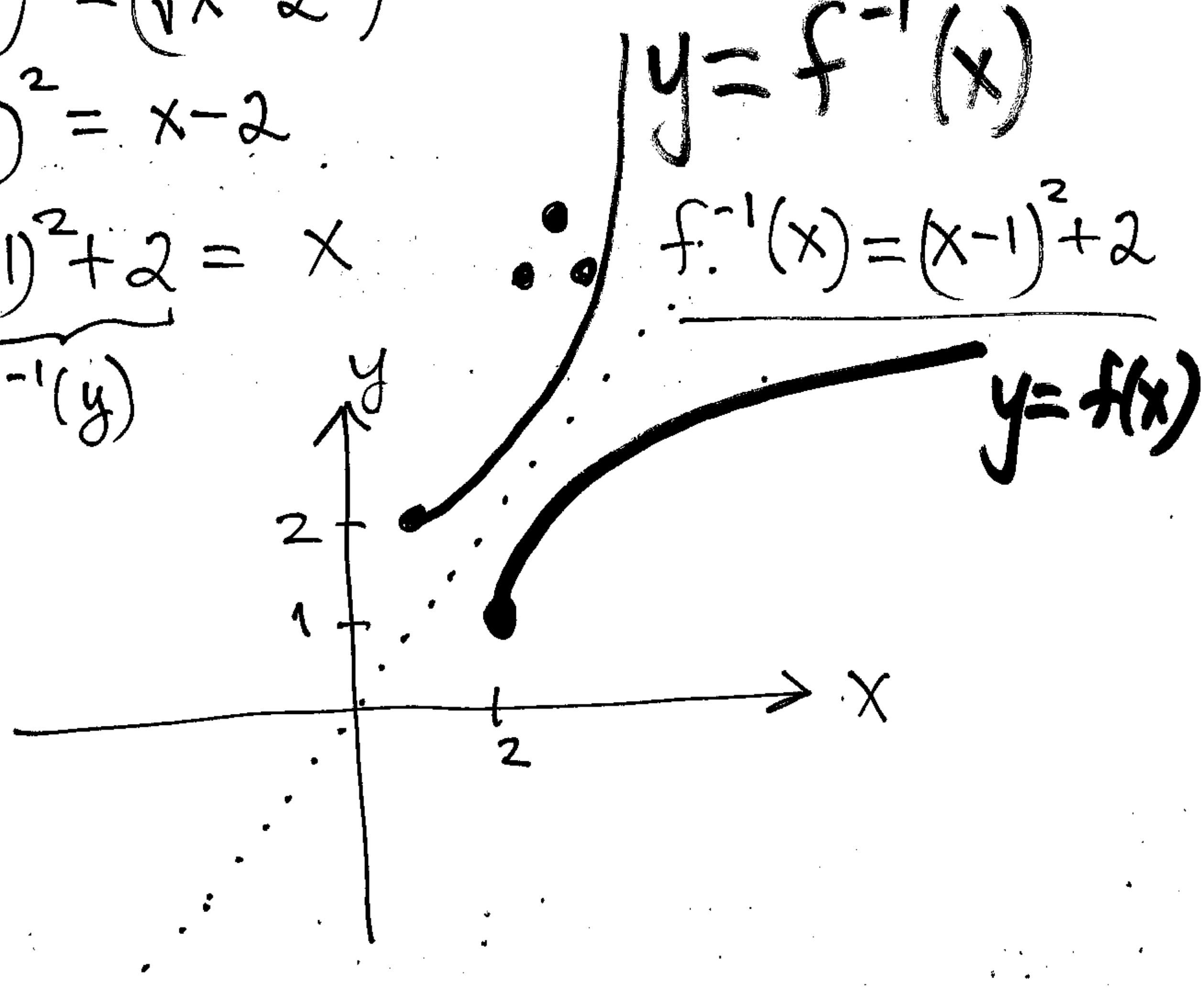
$$y = 1 + \sqrt{x-2}$$

$$(y-1)^2 = \left(\sqrt{x-2}\right)^2$$

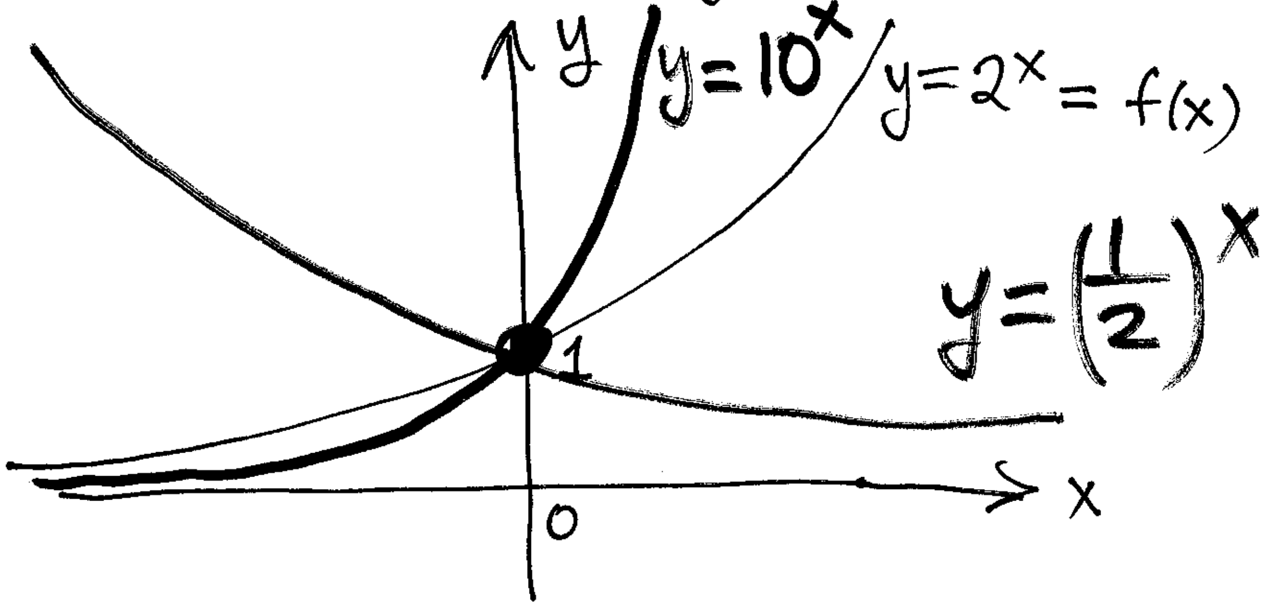
$$(y-1)^2 = x-2$$

$$\underbrace{(y-1)^2 + 2}_{f^{-1}(y)} = x$$

$\therefore$ $\underline{f^{-1}(x) = (x-1)^2 + 2}$



Exponential Functions: $y = f(x) = b^x$ $(b > 0,\ b \neq 1)$



$$y = \left(\frac{1}{2}\right)^x = 2^{-x} = f(-x)$$

Natural Exponential Function: $\boxed{f(x) = e^x}$

$e = 2.718\ldots$

All b^x are 1-1, hence have inverses:

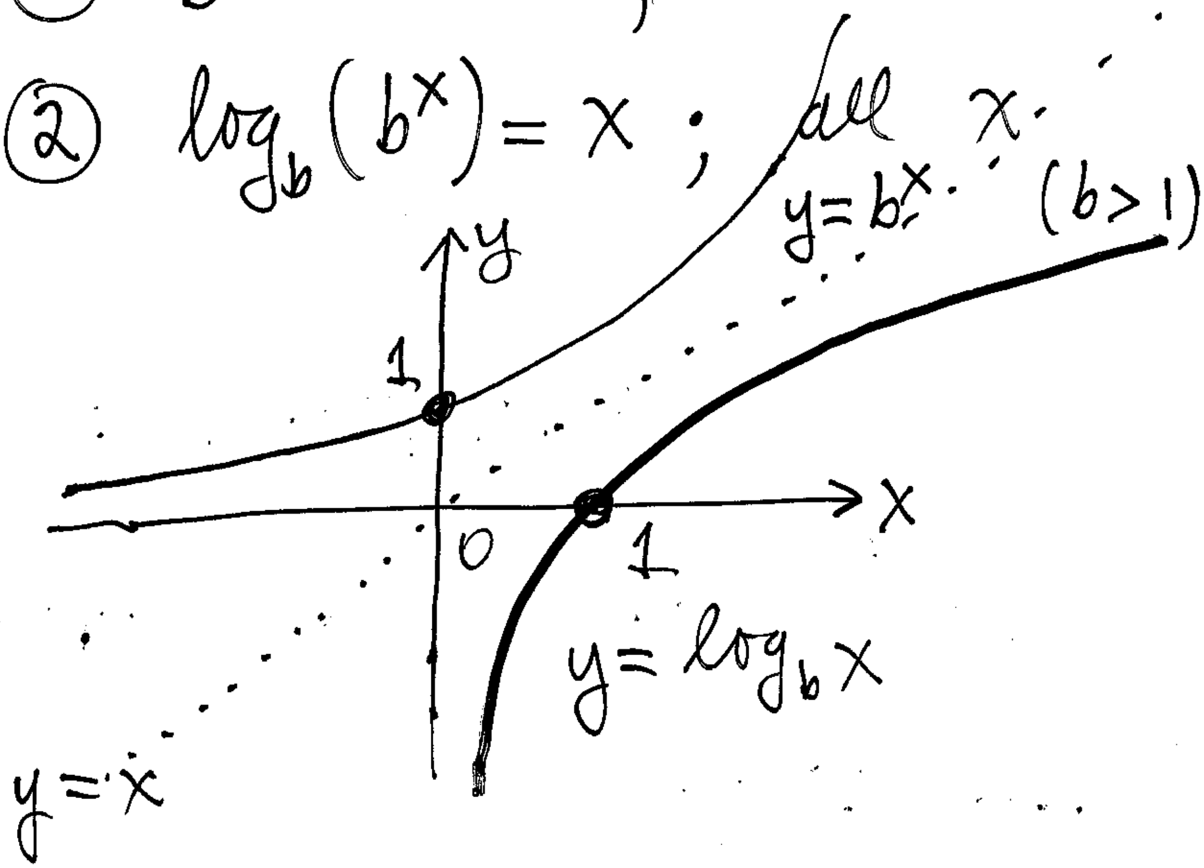
$\boxed{y = \log_b x}$ is the inverse of $y = b^x$.

(i.e. $y = \log_b x \Leftrightarrow b^y = x$)

Inverse Relations

① $b^{\log_b x} = x$; $x > 0$

② $\log_b(b^x) = x$; all x.



$\log_{10} x = \log x$ "common" logarithm

$\log_e x = \ln x$ "natural" logarithm

Law of Logarithms

① $\log_b(xy) = \log_b x + \log_b y$

② $\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$

③ $\log_b(x^a) = a \log_b x$