Lesson 10

§3.3 – Rules of Differentiation

$$\frac{d}{dx}\left(cf(x)\right) = \lim_{h \to 0} \frac{cf(x+h) - cf(x)}{h}$$

$$= c \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = c f'(x)$$

$$\frac{d}{dx}\left(x^3\right) = \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h}$$

$$= \lim_{h \to 0} \frac{(\cancel{x^3} + 3x^2h + 3xh^2 + h^3) - \cancel{x^3}}{h}$$

$$= \lim_{h \to 0} \left(3x^2 + 3xh + h^2\right) = 3x^2$$

Basic Differentiation Rules

(1) Constant Rule: $\frac{d(c)}{dx} = 0$

(2) Constant Multiple Rule: $\frac{d}{dx}(cf(x)) = cf'(x)$

(3) Sum Rule: $\frac{d}{dx}(f(x)+g(x)) = f'(x)+g'(x)$

(4) Difference Rule: $\frac{d}{dx}(f(x)-g(x)) = f'(x)-g'(x)$

(5) Power Rule: $\frac{d(x^n)}{dx} = nx^{n-1}$ for $n=1,2,3,\cdots$

$\boxed{\text{Ex1}}$ Let $f(x) = 12x^5 - 5x^3 + 1$

(a) Where is slope of tangent line 0?

Soln: $m_{tan} = f'(x) = 12(5x^4) - 5(3x^2) + 0$

$$= 60x^4 - 15x^2$$

$$= 15x^2(4x^2-1) = 0$$

$\Rightarrow x = 0$

$x = \pm\frac{1}{2}$

(b) Find equation of tangent line and normal line at $x = 1$.

Soln: At $x = 1 \Rightarrow m_{tan} = f'(1) = 45$

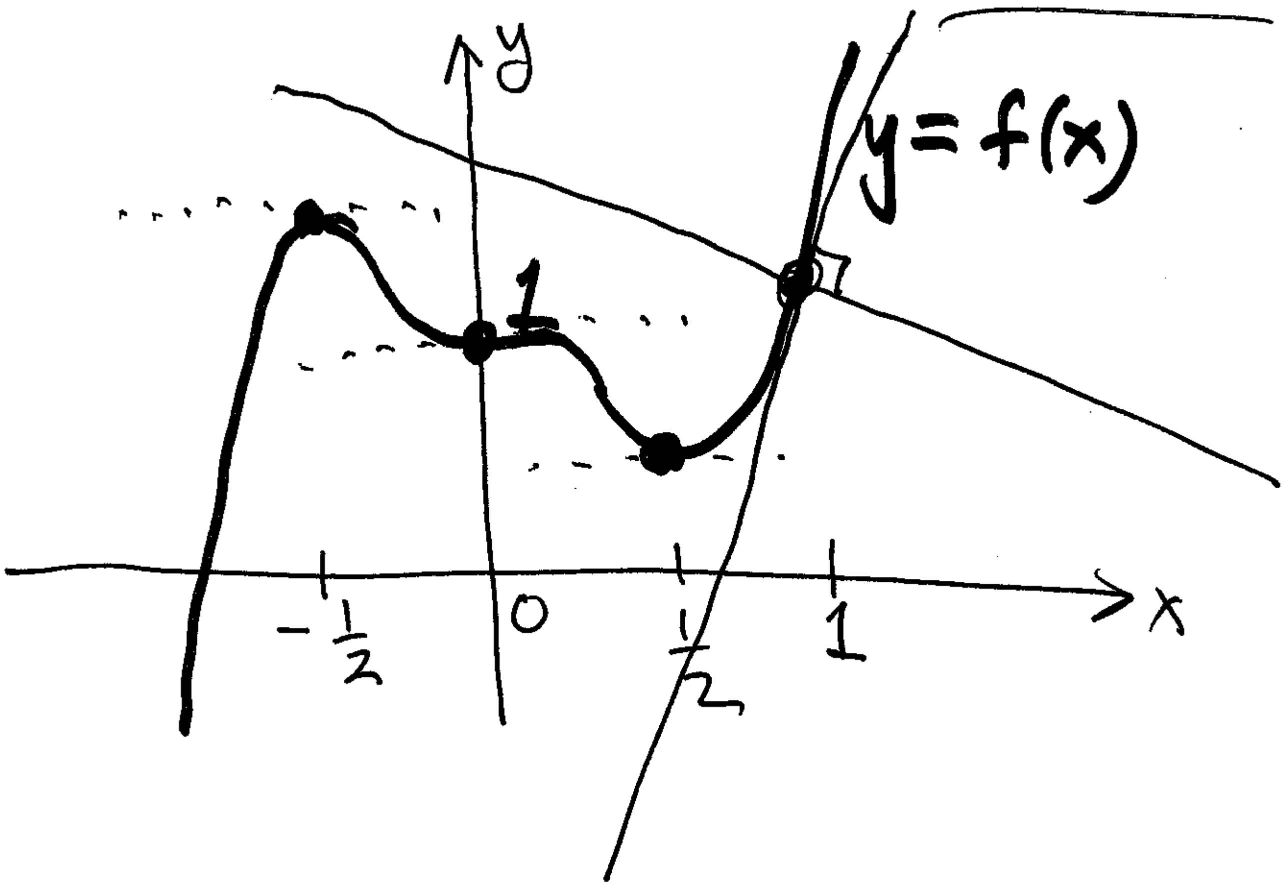
$\therefore y = f(1) = 8$

Eqn of tangent line: $y - 8 = 45(x - 1)$

$y = 45x - 37$ ✓

Eqn Normal line: $m_{normal} = -\frac{1}{m_{tan}} = -\frac{1}{45}$

$\therefore \boxed{y - 8 = -\frac{1}{45}(x - 1)}$ $\quad y = -\frac{1}{45}x + \frac{361}{45}$



Def: The number e is the number satisfying $\lim_{h\to 0} \frac{e^h - 1}{h} = 1$.

Hence $e = 2.71828\ldots$

Note: let $f(x) = e^x$ then

$e^x \cdot e^h$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{e^{x+h} - e^x}{h}$$

$$= e^x \lim_{h\to 0} \frac{e^h - 1}{h} = e^x$$

Thm: $\boxed{\frac{d(e^x)}{dx} = e^x}$

For eg: If $f(x) = 4e^x + x^2 + e^2$ find $f'(\ln 3)$.

$\therefore f'(x) = 4(e^x) + 2x + 0$

$$f'(\ln 3) = 4e^{\ln 3} + 2(\ln 3)$$

$$= 4(3) + 2\ln 3$$

$$= 12 + 2\ln 3$$

Higher Order Derivatives

Given $y = f(x)$

1st derivative: y', $\frac{dy}{dx}$, $f'(x)$

2nd derivative:

$$y'' = (y')' = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2}$$ Leibnitz!

etc ...

$\boxed{\text{Ex2}}$ Where is $m_{tan} = 0$ if

$$f(x) = (x^2-4)(2x+1)\ ?$$

$$f(x) = 2x^3 + x^2 - 8x - 4$$

$$m_{tan} = f'(x) = 6x^2 + 2x - 8 = 0$$

$$\Rightarrow \quad x = \frac{-2 \pm \sqrt{4 - 4(-48)}}{2(6)}$$

$$\Rightarrow \quad \underline{x = 1}, \quad \underline{x = -4/3}$$