Lesson 10

§3.3 - Rules of Differentiation

$$\frac{d(c f(x))}{dx} = \lim_{h \to 0} \frac{c f(x+h) - c f(x)}{h}$$

$$= c \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = c f'(x)$$

$$\frac{d(x^3)}{dx} = \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^3} + 3x^2 h + 3x h^2 + h^3 - \cancel{x^3}}{h}$$

$$= \lim_{h \to 0} (3x^2 + 3x h + h^2) = 3x^2$$

Basic Differentiation Rules

(1) <u>Constant Rule:</u> $\frac{d(c)}{dx} = 0$

(2) <u>Constant Multiple:</u> $\frac{d(cf(x))}{dx} = c f'(x)$

(3) <u>Sum Rule:</u> $\frac{d}{dx}\left(f(x) + g(x)\right) = f'(x) + g'(x)$

(4) <u>Difference Rule:</u> $\frac{d}{dx}\left(f(x) - g(x)\right) = f'(x) - g'(x)$

(5) <u>Power Rule:</u> $\frac{d(x^n)}{dx} = n x^{n-1}$ for $n = 1, 2, 3, \ldots$

$\boxed{\text{Ex 1}}$ Let $f(x) = 12x^5 - 5x^3 + 1$

(a) Where is slope of tangent line 0 ?

<u>Soln:</u> $m_{tan} = f'(x) = 12(5x^4) - 5(3x^2) + 0$

$$= 60x^4 - 15x^2 = 0$$

$$= 15x^2(4x^2 - 1) = 0$$

$x = 0$ $\qquad$ $x = \pm \frac{1}{2}$

(b) Find equation of tangent line and normal line at $x = 1$:

Soln: $\underbrace{x=1}$, $m_{tan} = f'(1) = 45$

$\therefore y = f(1) = 8$

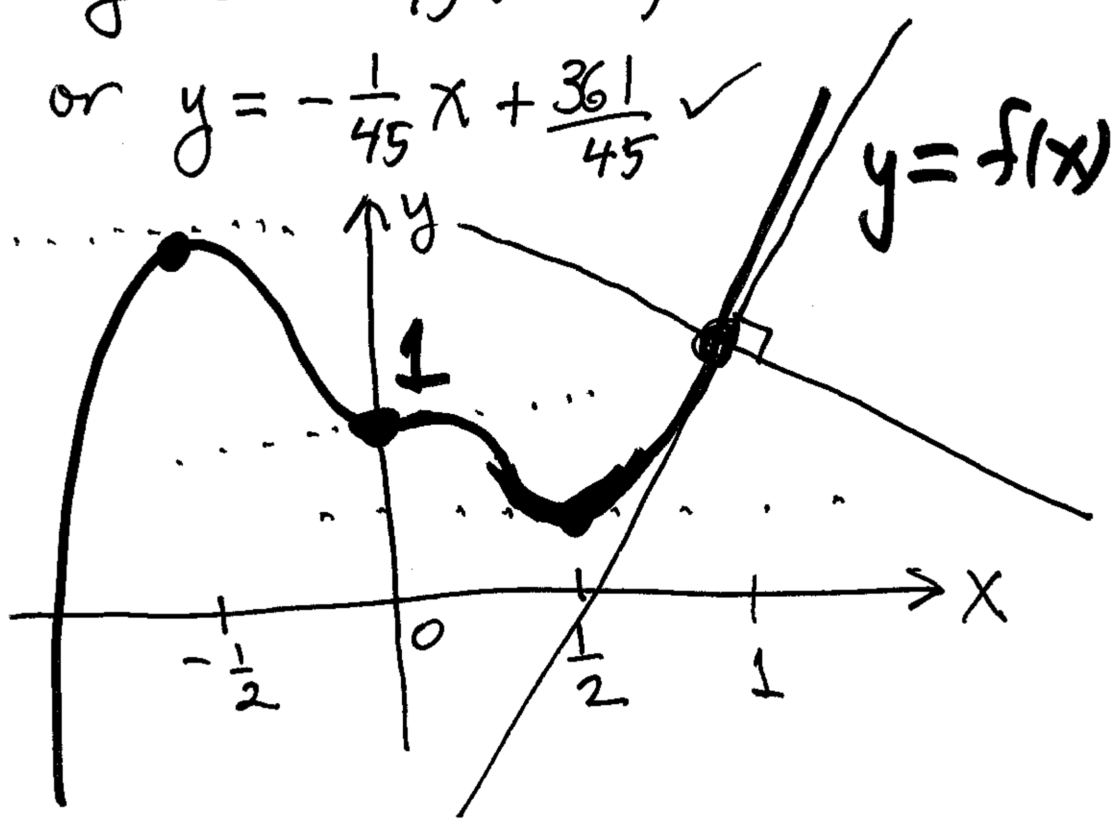
$\therefore$ Eqn of tangent line: $y - 8 = 45(x-1)$ ✓

$y = 45x - 37$ ✓

Eqn of normal line: $m_{normal} = -\dfrac{1}{m_{tan}} = -\dfrac{1}{45}$

$\therefore y - 8 = -\frac{1}{45}(x-1)$ ✓

or $y = -\frac{1}{45}x + \frac{361}{45}$ ✓



Def: The number e is the number satisfying

$$\lim_{h\to 0} \frac{e^h - 1}{h} = 1$$

Hence $e = 2.71828\ldots$

Note: let $f(x) = e^x$

$$e^x \cdot e^h$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{e^{x+h} - e^x}{h}$$

$$= \lim_{h\to 0} e^x\left(\frac{e^h - 1}{h}\right) = e^x \lim_{h\to 0} \frac{e^h - 1}{h} = e^x$$

Thm: $\boxed{\dfrac{d(e^x)}{dx} = e^x}$

For eg: Find $f'(\ln 3)$ if $f(x) = 4e^x + x^2 + e^2$

$$f'(x) = 4e^x + 2x + 0$$

$$f'(\ln 3) = 4e^{\ln 3} + 2(\ln 3)$$

$$= 4(3) + 2\ln 3$$

$$= 12 + 2\ln 3$$

Higher Order Derivatives: Given $y = f(x)$

1st derivative: $y' = \frac{df}{dx} = \frac{dy}{dx}$

2nd derivative: $y'' = (y')' = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2}$

$y''' = \frac{d^3y}{dx^3}$, etc . . . .

Leibnitz

$\boxed{Ex2}$ Where is $m_{tan} = 0$ if $f(x) = (x^2-4)(2x+1)$?

$f(x) = 2x^3 + x^2 - 8x - 4$

$\therefore f'(x) = 6x^2 + 2x - 8 = 0$