Lesson 11

§3.4 - Product + Quotient Rules

$$\frac{d}{dx}\{f(x)g(x)\} = \lim_{h\to 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h)[g(x+h) - g(x)] + g(x)[f(x+h) - f(x)]}{h}$$

$$= f(x)g'(x) + g(x)f'(x)$$

Product Rule: If f, g are differentiable

then $\boxed{\frac{d}{dx}\{f(x)g(x)\} = f(x)g'(x) + g(x)f'(x)}$

Ex 1 Compute $\frac{d^2y}{dx^2}$ if $y = (x^2 + e^x)(4x - 1)$

Soln: $y' = (x^2+e^x)(4) + (4x-1)(2x+e^x)$ $\boxed{2}$

$y'' = 4(2x+e^x) + (4x-1)(2+e^x) + (2x+e^x)(4)$

<u>Quotient Rule:</u> If f, g are differentiable

then
$$\boxed{\frac{d}{dx}\left\{\frac{f(x)}{g(x)}\right\} = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}}$$

provided $g(x) \neq 0$

Reason:

$$\frac{d}{dx}\left\{\frac{f(x)}{g(x)}\right\} = \lim_{h \to 0} \frac{\dfrac{f(x+a)}{g(x+a)} - \dfrac{f(x)}{g(x)}}{h}$$

$$= \lim_{h \to 0} \frac{f(x+a)g(x) - g(x+h)f(x)}{h\, g(x+h)\, g(x)}$$

$$= \lim_{h \to 0} \frac{g(x)\left[f(x+h) - f(x)\right] - f(x)\left[g(x+h) - g(x)\right]}{h\, g(x+h)\, g(x)}$$

$$= \frac{g(x) f'(x) - f(x) g'(x)}{(g(x))^2}$$

$\boxed{\text{Ex 2}}$ Find derivative:

(a) $y = \dfrac{2x^3+1}{x^2-e^x}$

$$\Rightarrow y' = \frac{(x^2-e^x)(6x^2) - (2x^3+1)(2x-e^x)}{(x^2-e^x)^2}$$

(b) $g(x) = \dfrac{1}{x^p}$, $p = 1, 2, 3, \ldots$

$$\Rightarrow g'(x) = \frac{x^p(0) - (1)(p x^{p-1})}{(x^p)^2} = \frac{-p x^{p-1}}{x^{2p}}$$

$$= -p x^{-p-1} \checkmark$$

$g(x) = x^{-p}$ Hence

General Power Rule: If n is any real number

then $\boxed{\dfrac{d(x^n)}{dx} = n x^{n-1}}$

Ex3 Compute $f'(x)$

ⓐ $f(x) = 4x + \frac{1}{\sqrt{x}} = 4x + \frac{1}{x^{1/2}} = 4x + x^{-\frac{1}{2}}$

$\Rightarrow f'(x) = 4 + \left(-\frac{1}{2}x^{-3/2}\right)$ ✓

$= 4 - \frac{1}{2}x^{-3/2}$ ✓

ⓑ $f(x) = \dfrac{x^2 e^x}{1 + \sqrt[3]{x}}$ $\leftarrow x^{1/3}$

$$\Rightarrow f'(x) = \frac{\left(1 + x^{1/3}\right)\left[x^2 e^x + e^x(2x)\right] - \left(x^2 e^x\right)\left[\frac{1}{3}x^{-2/3}\right]}{\left(1 + \sqrt[3]{x}\right)^2}$$

$\boxed{\text{Ex4}}$ Compute $\lim_{x \to 2} \frac{3x^5 - 3(32)}{x-2} = f'(2)$

where $f(x) = 3x^5$.

$$= 15x^4 \Big|_{x=2}$$

$$= \underline{\underline{240}}$$

$\boxed{\text{Ex5}}$ The # of students (in hundreds) with the flu after t months is

$$F(t) = 4t^2 - \frac{1}{3}t^3 + 1.$$

Find rate of change of $F(t)$ when $t = 5$.

<u>Soln</u>: $F'(t) = 8t - t^2$

$$F'(5) = 15$$