Lesson 17

§3.10 Derivatives of Inverse Trig Fcns

Fall Final Exam Fall 2018 #11

If $f(x)=\log_{10}x$, $f'(e)=?$

let $y=\log_{10}x \iff 10^y=x$ Use Log. Diff.

$\Rightarrow \ln(10^y)=\ln x$

$y\ln 10=\ln x$

$\therefore \left(\frac{dy}{dx}\right)\ln 10=\frac{1}{x} \Rightarrow \frac{dy}{dx}=\frac{1}{x\ln 10}$

$\therefore f'(e)=\frac{1}{e\ln 10}$ ✓

Ex1 Find $\frac{dy}{dx}$ if

(a) $\sin y = x$ Use Implicit Diff

$$\frac{d}{dx}\{\sin y\} = \frac{d}{dx}\{x\}$$

$$(\cos y)\left(\frac{dy}{dx}\right) = 1 \Rightarrow \frac{dy}{dx} = \frac{1}{\cos y}$$

If we restrict $-\frac{\pi}{2} \le y \le \frac{\pi}{2} \Rightarrow y = \sin^{-1} x$

$$\therefore \frac{d\{\sin^{-1} x\}}{dx} = \frac{1}{\cos y}$$

$$= \frac{1}{\sqrt{1-x^2}}$$

; Recall $\cos^2 y + \sin^2 y = 1$

$$\cos y = \pm\sqrt{1-\sin^2 y}$$

$\cos y \ge 0$ in $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

(b) $\tan y = x$ Use Implicit Diff.

$$\Rightarrow (\sec^2 y)\left(\frac{dy}{dx}\right) = 1 \Rightarrow \frac{dy}{dx} = \frac{1}{\sec^2 y}$$

If let $-\frac{\pi}{2} < y < \frac{\pi}{2} \Rightarrow y = \tan^{-1} x$

$\therefore$ $\dfrac{d(\tan^{-1} x)}{dx} = \dfrac{1}{\sec^2 y}$; recall $1 + \tan^2 y = \sec^2 y$

$$= \frac{1}{1 + x^2}$$

Thm (Derivative of Inverse Trig Fcns)

$\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$	$\frac{d(\cos^{-1}x)}{dx} = -\frac{1}{\sqrt{1-x^2}}$ $(-1<x<1)$
$\frac{d(\tan^{-1}x)}{dx} = \frac{1}{1+x^2}$	$\frac{d(\cot^{-1}x)}{dx} = -\frac{1}{1+x^2}$ $(-\infty<x<\infty)$
$\frac{d(\sec^{-1}x)}{dx} = \frac{1}{\lvert x\rvert\sqrt{x^2-1}}$	$\frac{d(\csc^{-1}x)}{dx} = -\frac{1}{\lvert x\rvert\sqrt{x^2-1}}$ $(\lvert x\rvert>1)$

Ex2 Find derivative

(a) $y = \sin^{-1}(x^2-3)$

$\therefore\ y' = \frac{1}{\sqrt{1-(x^2-3)^2}}(2x)$ ✓

$y' = \frac{2x}{\sqrt{6x^2-x^4-8}}$ ✓

(b) Find $\frac{dy}{dx}$ if $x^2y^3 = \tan^{-1}(2y)$ [5]

Use Implicit Diff.:

$$x^2\left\{3y^2\frac{dy}{dx}\right\} + y^3\{2x\} = \frac{1}{1+(2y)^2}\cdot\left(2\frac{dy}{dx}\right)$$

$$\vdots$$

$$\frac{dy}{dx} = \frac{-2xy^3}{\left[3x^2y^2 - \frac{2}{1+4y^2}\right]}$$ ✓

(c) $f(x) = (\sin^{-1}x)^x$ Use Log Diff.

$$\Rightarrow \ln f(x) = x\ln(\sin^{-1}x)$$

$$\frac{1}{f(x)}f'(x) = x\left[\frac{1}{\sin^{-1}x}\,\frac{1}{\sqrt{1-x^2}}\right] + \ln(\sin^{-1}x)$$ diff. w.r.t. to x

$$\Rightarrow f'(x) = f(x)\left\{x\left[\frac{1}{\sin^{-1}x}\,\frac{1}{\sqrt{1-x^2}}\right] + \ln(\sin^{-1}x)\right\}$$

Thm: (Derivative of Inverse Fcn)
If f is diff. and has inverse on I, x_0 is a pt of I, and $f'(x_0) \neq 0$

$$\boxed{\left(f^{-1}\right)'(f(x_0)) = \frac{1}{f'(x_0)}}$$