Lesson 18 ← not on Exam

§3.11 – Related Rates

Sphere is heated so that its volume is increasing at a rate of 2 mm^3/hr.
How fast is radius changing when radius is 8 mm ?

given rate: $\frac{dV}{dt} = 2\ mm^3/hr$

desired rate: $\frac{dr}{dt}$ when $r = 8\ mm$

Equation: $\boxed{V = \frac{4}{3}\pi r^3}$

$$\therefore \frac{dV}{dt} = \frac{d}{dt}\left\{\frac{4}{3}\pi r^3\right\} = \frac{4}{3}\pi\left(3r^2\frac{dr}{dt}\right)$$

$$\Rightarrow \left.\frac{dr}{dt}\right|_{r=8} = \frac{1}{4\pi r^2}\left.\frac{dV}{dt}\right|_{r=8} = \frac{1}{4\pi(8)^2}(2)$$

$$= \frac{1}{128\pi}\ mm/hr \checkmark$$

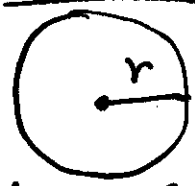
Related Rates Method

1. Read problem several times.
2. Draw a picture and label with variables
3. Write down
   - given rate
   - desired rate
   - Equation relating the variables
4. Diff. Equation w.r.t. t
5. Solve for desired rate

Remarks

(a) If $f'(t) > 0 \Rightarrow f$ is increasing
If $f'(t) < 0 \Rightarrow f$ is decreasing

(b) Formulas

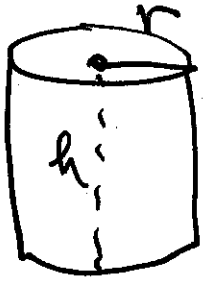


$A = \pi r^2$

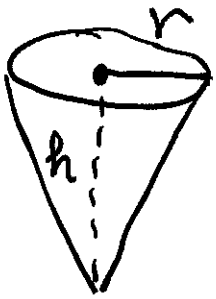
$C = 2\pi r$

$V = \frac{4}{3}\pi r^3$

$S = 4\pi r^2$

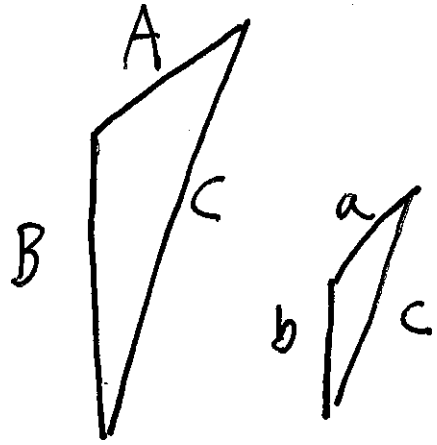


$V = \pi r^2 h$



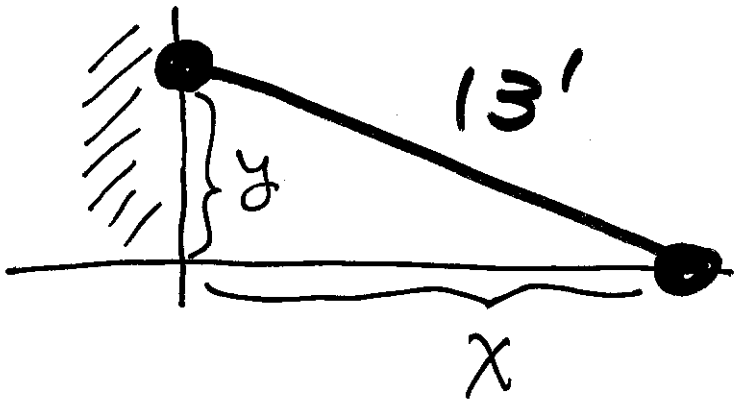
$V = \frac{1}{3}\pi r^2 h$

(c) Similar triangles



$$\frac{A}{B} = \frac{a}{b}$$

RR-01



given rate: $\frac{dx}{dt} = 3$ ft/sec

desired rate: $\frac{dy}{dt}$ when $x = 5$

Equation: $\boxed{x^2 + y^2 = 13^2}$ ⊛

$$\frac{d}{dt}\{x^2 + y^2\} = \frac{d}{dt}\{13^2\}$$

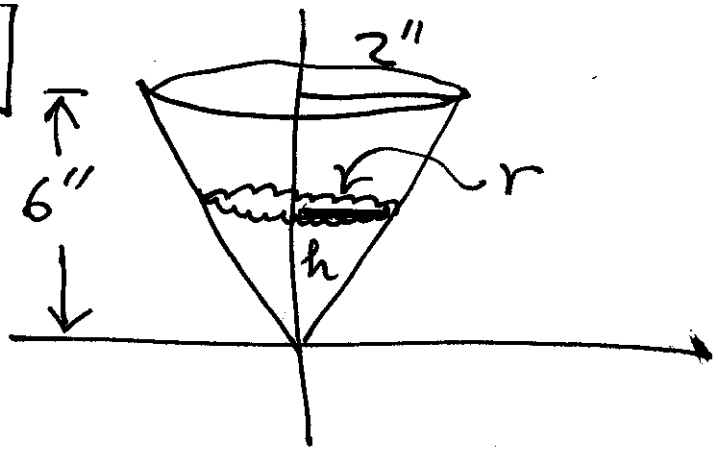
$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0 \Rightarrow \frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt}$$

When $x = 5$ ⊛ $\Rightarrow y = 12$

$$\therefore \left.\frac{dy}{dt}\right|_{x=5} = -\frac{5}{12}(3) = -\frac{5}{4} \text{ ft/sec} \checkmark$$

∴ Top slides <u>down</u> at $\frac{5}{4}$ ft/sec ✓

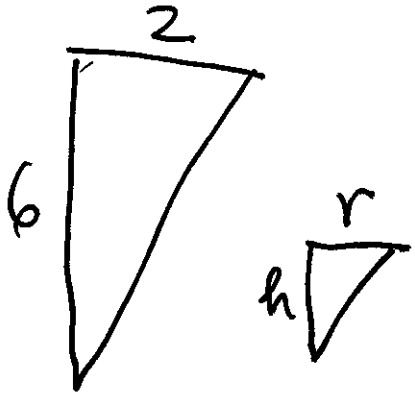
RR-02



given rate: $\frac{dV}{dt} = \frac{2}{3}$ in^3/sec

desired rate: $\frac{dh}{dt}$ when $h = 4$ in.

Equation: $V = \frac{1}{3}\pi r^2 h$



$\Rightarrow \frac{2}{6} = \frac{r}{h}$

$\frac{1}{3}h = r$

$\therefore V = \frac{1}{3}\pi\left(\frac{1}{3}h\right)^2 h$

$$\boxed{V = \frac{\pi}{27}h^3}$$

$$\frac{dV}{dt} = \frac{\pi}{27}\left(3h^2 \frac{dh}{dt}\right)$$

$$\left.\frac{dh}{dt}\right|_{h=4} = \frac{9}{\pi h^2}\left(\frac{dV}{dt}\right)\Bigg|_{h=4}$$

$$= \frac{9}{\pi(4)^2}\left(\frac{2}{3}\right) = \frac{3}{8\pi} \text{ in/sec} \checkmark$$