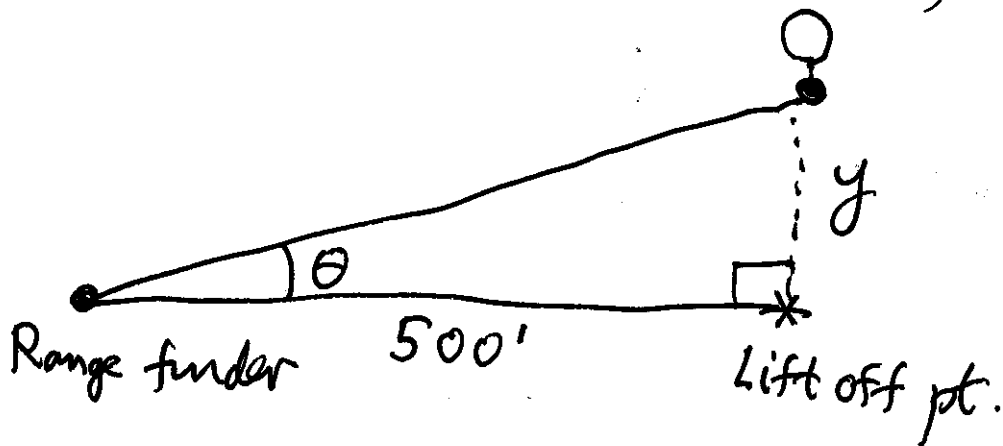


Exam #2
Thursday, Oct 21
Lessons 9-17

Lesson 19
Related Rates (cont'd)

①

RR-09



given rate: $\frac{d\theta}{dt} = 0.14$ radians/min when $\theta = \frac{\pi}{4}$

desired rate: $\frac{dy}{dt}$ when $\theta = \frac{\pi}{4}$

Equation:

$$\tan \theta = \frac{y}{500}$$

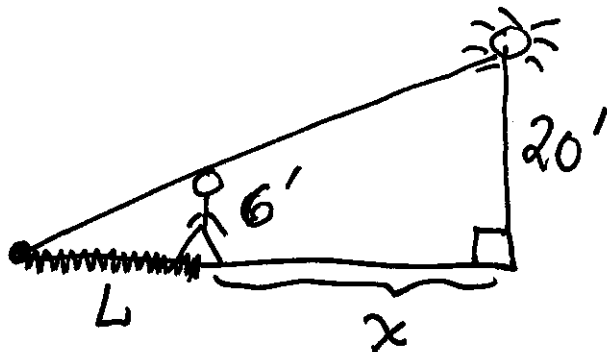
diff. w.r.t. t

$$(\sec^2 \theta) \frac{d\theta}{dt} = \frac{1}{500} \left(\frac{dy}{dt} \right)$$

$$\therefore \left. \frac{dy}{dt} \right|_{\theta = \frac{\pi}{4}} = 500 (\sec^2 \theta) \left. \frac{d\theta}{dt} \right|_{\theta = \frac{\pi}{4}} = (500) \left(\frac{2}{\sqrt{2}} \right)^2 (0.14) = 140 \text{ ft/min}$$

RR-03

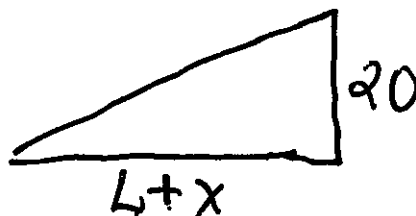
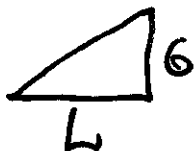
(2)



given rate: $\frac{dx}{dt} = -5 \text{ ft/sec}$

desired rate: $\frac{dL}{dt}$ when $x = 24$

[Equation]:



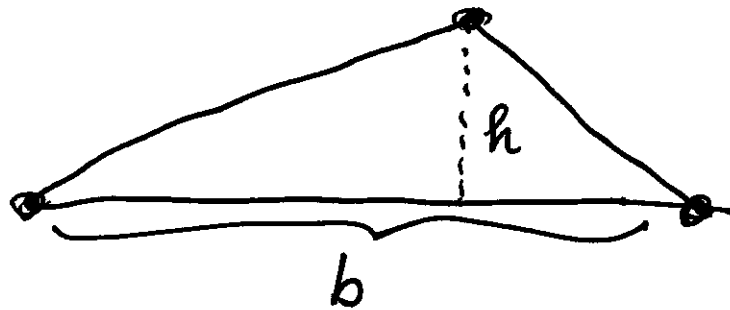
Similar Δ 's $\Rightarrow \frac{L}{6} = \frac{L+x}{20}$

$\Rightarrow \boxed{L = \frac{3}{7}x}$ \leftarrow this is equation!

$\therefore \frac{dL}{dt} \Big|_{x=24} = \frac{3}{7} \frac{dx}{dt} = \left(\frac{3}{7}\right)(-5) = -\frac{15}{7} \text{ ft/sec}$

Hence length of shadow decreases at $\frac{15}{7} \text{ ft/sec}$

RR-11



③

given rate: $\frac{dh}{dt} = 1 \text{ cm/min}$ and $\frac{dA}{dt} = 2 \text{ cm}^2/\text{min}$

desired rate: $\frac{db}{dt}$ when $h = 10 \text{ cm}$, $A = 100 \text{ cm}^2$

Equation: $A = \frac{1}{2}bh$ (*) diff. w.r.t. t

$$\Rightarrow \frac{dA}{dt} = \frac{1}{2} \left(b \frac{dh}{dt} + h \frac{db}{dt} \right)$$

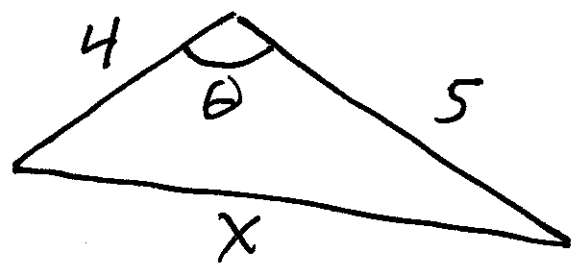
$$\therefore \frac{db}{dt} = \frac{2 \frac{dA}{dt} - b \frac{dh}{dt}}{h}$$

When $h = 10$,
 $A = 100$
(*) $\Rightarrow b = 20$

$$\left. \frac{db}{dt} \right|_{h=10} = \frac{2(2) - (20)(1)}{10} = -\frac{8}{5} \text{ cm/min}$$

Base is decreasing at $\frac{8}{5} \text{ cm/min}$

RR-14



given rate: $\frac{d\theta}{dt} = \frac{1}{10}$ radians/min

desired rate: $\frac{dx}{dt}$ when $\theta = \frac{\pi}{3}$ (60°)

Equation: Law of Cosines

$$x^2 = 4^2 + 5^2 - 2(4)(5)\cos\theta$$

$$x^2 = 41 - 40\cos\theta \quad (*)$$

$$\Rightarrow 2x \frac{dx}{dt} = (40\sin\theta) \frac{d\theta}{dt}$$

$$\frac{dx}{dt} = \frac{(20\sin\theta)}{x} \frac{d\theta}{dt}$$

when $\theta = \frac{\pi}{3}$ (*)
 $\Rightarrow x = \sqrt{21}$

$$\therefore \frac{dx}{dt} = \frac{1}{\sqrt{7}} \text{ cm/min}$$