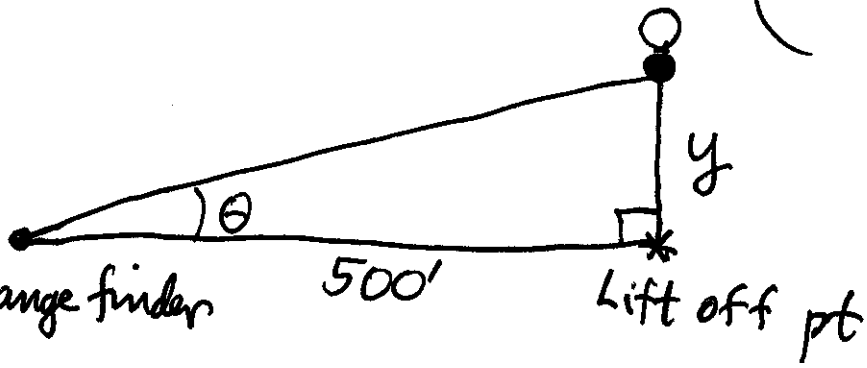
Exam #2
Thursday, Oct 21
Lessons 9-17

Lesson 19

Related Rates (cont'd)

RR-09



given rate: $\frac{d\theta}{dt} = 0.14$ radians/min, when $\theta = \frac{\pi}{4}$

desired rate: $\frac{dy}{dt}$ when $\theta = \frac{\pi}{4}$

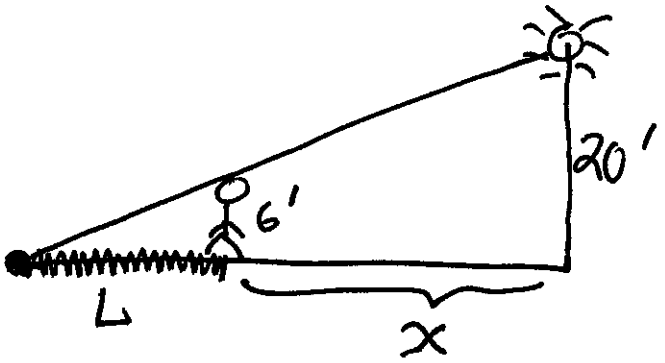
Equation: $\boxed{\tan\theta = \frac{y}{500}}$ diff. w.r.t. t

$$\Rightarrow (\sec^2\theta)\frac{d\theta}{dt} = \frac{1}{500}\frac{dy}{dt}$$

$$\Rightarrow \left.\frac{dy}{dt}\right|_{\theta=\frac{\pi}{4}} = 500(\sec^2\theta)\left.\frac{d\theta}{dt}\right|_{\theta=\frac{\pi}{4}}$$

$$= 500\left(\frac{2}{\sqrt{2}}\right)^2(0.14) = 140 \text{ ft/min} \checkmark$$

RR-03



given rate: $\frac{dx}{dt} = -5$ ft/sec

desired rate: $\frac{dL}{dt}$ when $x = 24$ ft.

Equation:

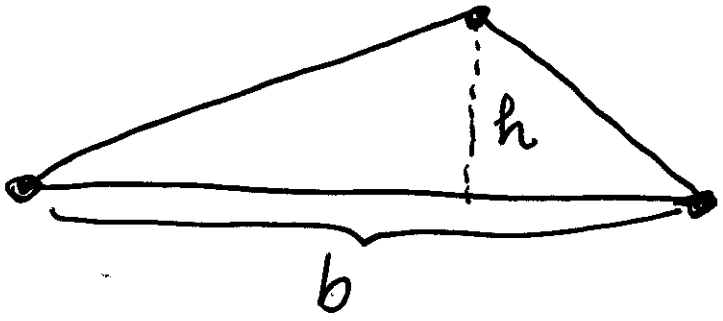
Similar Δ's $\Rightarrow \frac{L}{6} = \frac{L+x}{20}$

$\Rightarrow \boxed{L = \frac{3}{7}x}$ ← Equation!

$\Rightarrow \left.\frac{dL}{dt}\right|_{x=24} = \frac{3}{7}\left.\frac{dx}{dt}\right|_{x=24} = \left(\frac{3}{7}\right)(-5) = -\frac{15}{7}$ ft/sec

∴ length of shadow decreases at $\frac{15}{7}$ ft/sec

RR-11



given rate: $\frac{dh}{dt} = 1$ cm/min and $\frac{dA}{dt} = 2$ cm^2/min

desired rate: $\frac{db}{dt}$ when $h = 10$ cm, $A = 100$ cm^2

Equation: $\boxed{A = \frac{1}{2}bh}$ ⊛ diff. w.r.t. t

$$\frac{dA}{dt} = \frac{1}{2}\left(b\frac{dh}{dt} + h\frac{db}{dt}\right)$$

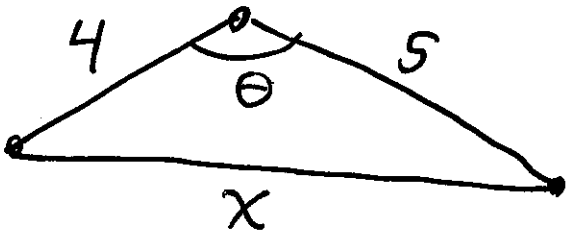
if $h = 10$, $A = 100$
⊛ $\Rightarrow 20 = b$

$$\therefore \frac{db}{dt} = \frac{2\frac{dA}{dt} - b\frac{dh}{dt}}{h}$$

$$\therefore \frac{db}{dt} = \frac{2(2) - (20)(1)}{10} = -8/5 \text{ cm/min}$$

base decreases at 8/5 cm/min

RR-14



given rate: $\frac{d\theta}{dt} = \frac{1}{10}$ radians/min

desired rate: $\frac{dx}{dt}$ when $\theta = \frac{\pi}{3}$ $(60°)$

Equation: Law of Cosines:

$$x^2 = 4^2 + 5^2 - 2(4)(5)\cos\theta$$

$$\boxed{x^2 = 41 - 40\cos\theta} \quad \circledast \text{ diff. w.r.t } t$$

$$2x\frac{dx}{dt} = (40\sin\theta)\frac{d\theta}{dt}$$

$$\frac{dx}{dt} = \left(\frac{20\sin\theta}{x}\right)\frac{d\theta}{dt}$$

$$= \frac{1}{\sqrt{7}}$$

when $\theta = \frac{\pi}{3}$

$\circledast \Rightarrow x = \sqrt{21}$