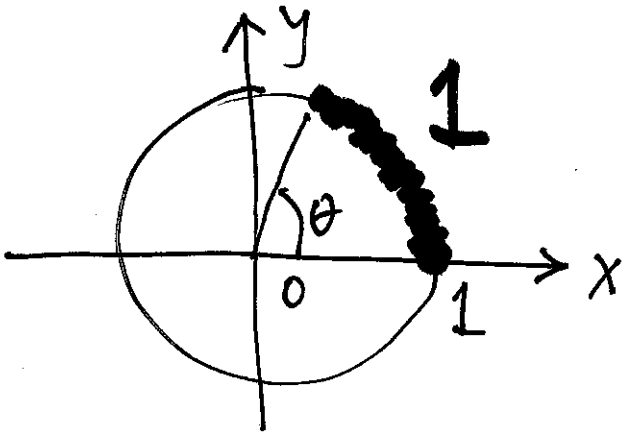
§1.4 Trig Functions (Review)

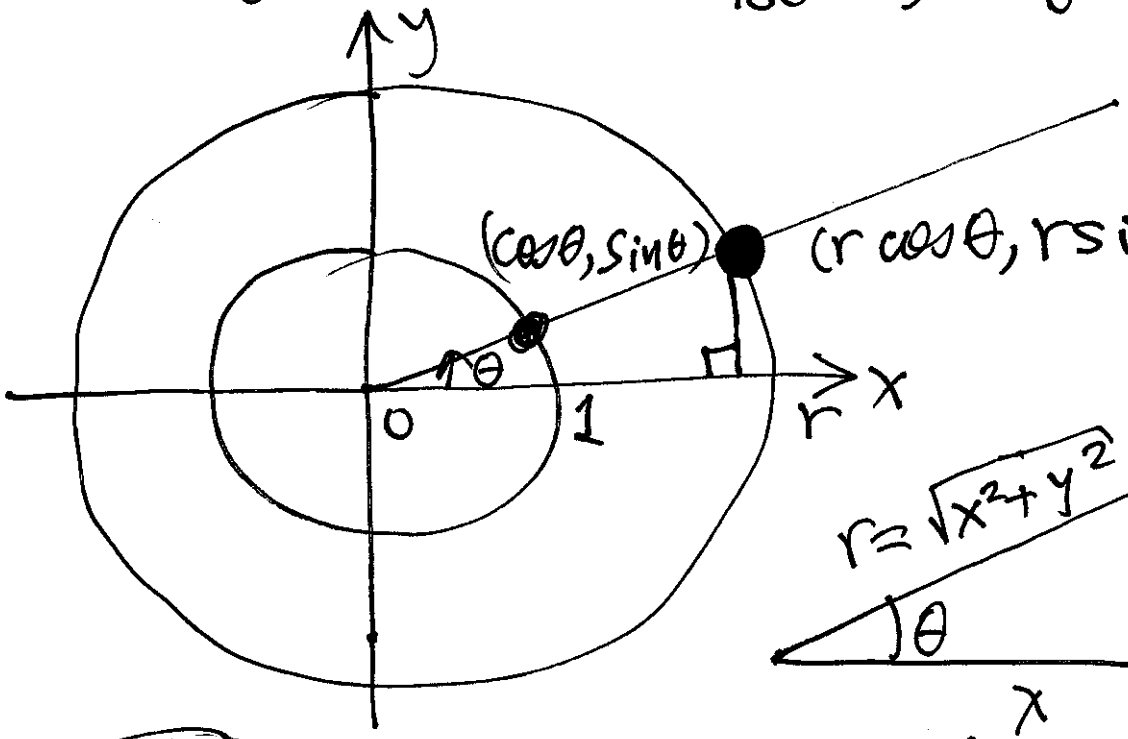


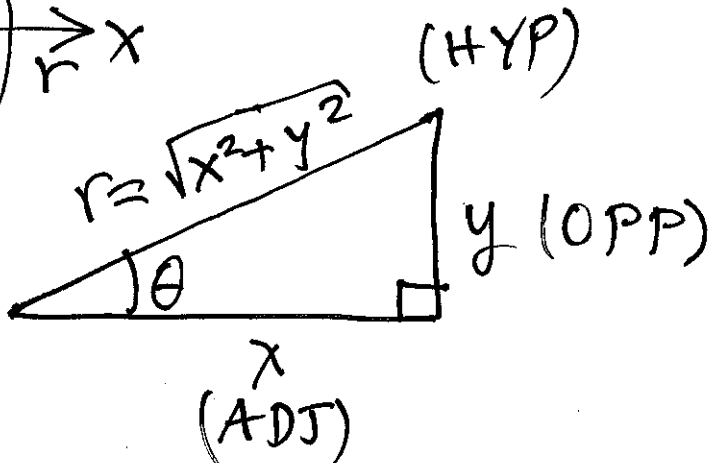
$\theta = 1$ radian

2π radians $= 360°$

$$\theta = \frac{\pi}{180} \times \text{degrees}$$

For eg $30°$ is $\theta = \frac{\pi}{180}(30) = \frac{\pi}{6}$ radians





$$x = r\cos\theta$$
$$y = r\sin\theta$$

$$\therefore \cos\theta = \frac{x}{r} = \frac{ADJ}{HYP}$$

$$\sin\theta = \frac{y}{r} = \frac{OPP}{HYP}$$

$$\tan\theta = \frac{y}{x}$$

$\sec\theta = \frac{1}{\cos\theta}$, $\csc\theta = \frac{1}{\sin\theta}$, $\cot\theta = \frac{1}{\tan\theta}$ [2]

Basic Table

θ	$\sin\theta$	$\cos\theta$
$0°$ (0)	0	1
$30°$ $\pi/6$	$1/2$	$\sqrt{3}/2$
$45°$ $\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$
$60°$ $\pi/3$	$\sqrt{3}/2$	$1/2$
$90°$ $\pi/2$	1	0

$\sin(\theta+2\pi) = \sin\theta$
$\cos(\theta+2\pi) = \cos\theta$ ← have period 2π

$\tan(\theta+\pi) = \tan\theta$ ← period π

$\cos(-\theta) = \cos\theta$ ← cos is an <u>even</u> function

$\sin(-\theta) = -\sin\theta$ ← sine is <u>odd</u> function

For eg $\sin\left(\frac{11\pi}{6}\right) = \sin\left(2\pi - \frac{\pi}{6}\right) = \sin\left(-\frac{\pi}{6}\right)$

$= -\sin\frac{\pi}{6} = -\frac{1}{2}$

Basic Identities:

[1] $\sin^2\theta + \cos^2\theta = 1$

[2] $\tan^2\theta + 1 = \sec^2\theta$

and $1 + \cot^2\theta = \csc^2\theta$

[3] $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos^2\theta = \frac{1+\cos 2\theta}{2}$, $\sin^2\theta = \frac{1-\cos 2\theta}{2}$

Ex1 Solve $1 - 2\cos x = 0$

Soln: $1 = 2\cos x \implies \frac{1}{2} = \cos x$

$x = \frac{\pi}{3}, \frac{\pi}{3}+2\pi, \frac{\pi}{3}+4\pi, \frac{\pi}{3}+6\pi, \ldots$

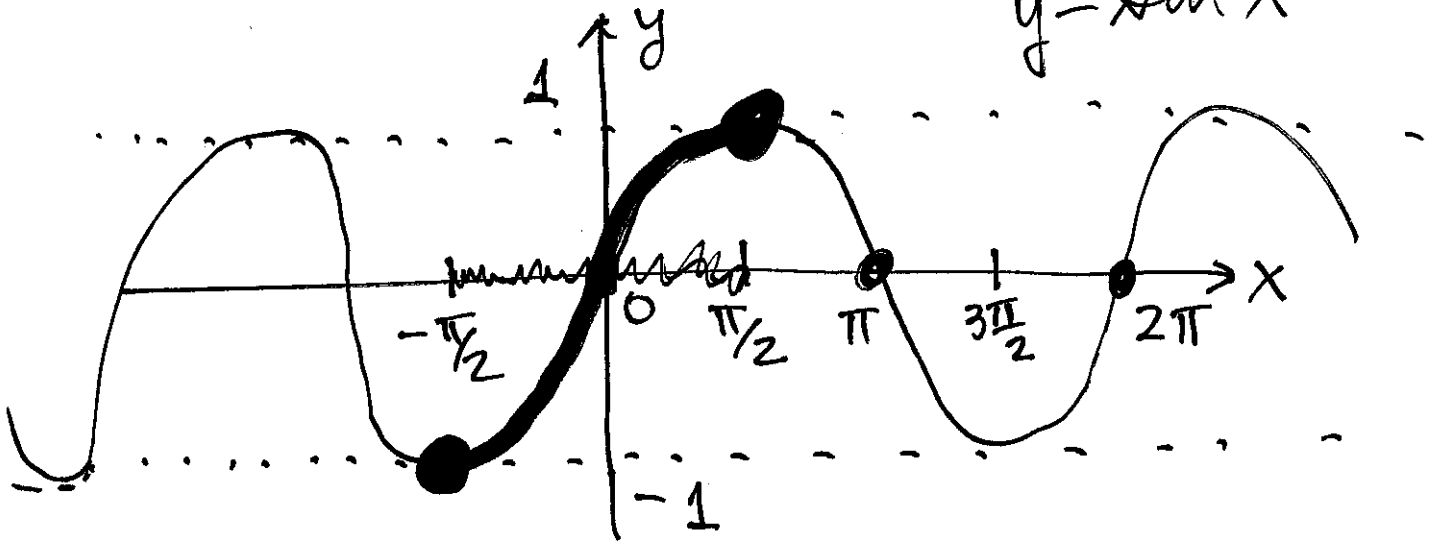
Also $x = \frac{\pi}{3}-2\pi, \frac{\pi}{3}-4\pi, \frac{\pi}{3}-6\pi, \ldots$

$\therefore$ $x = \frac{\pi}{3} + 2\pi k$, $k = 0, \pm 1, \pm 2, \ldots$

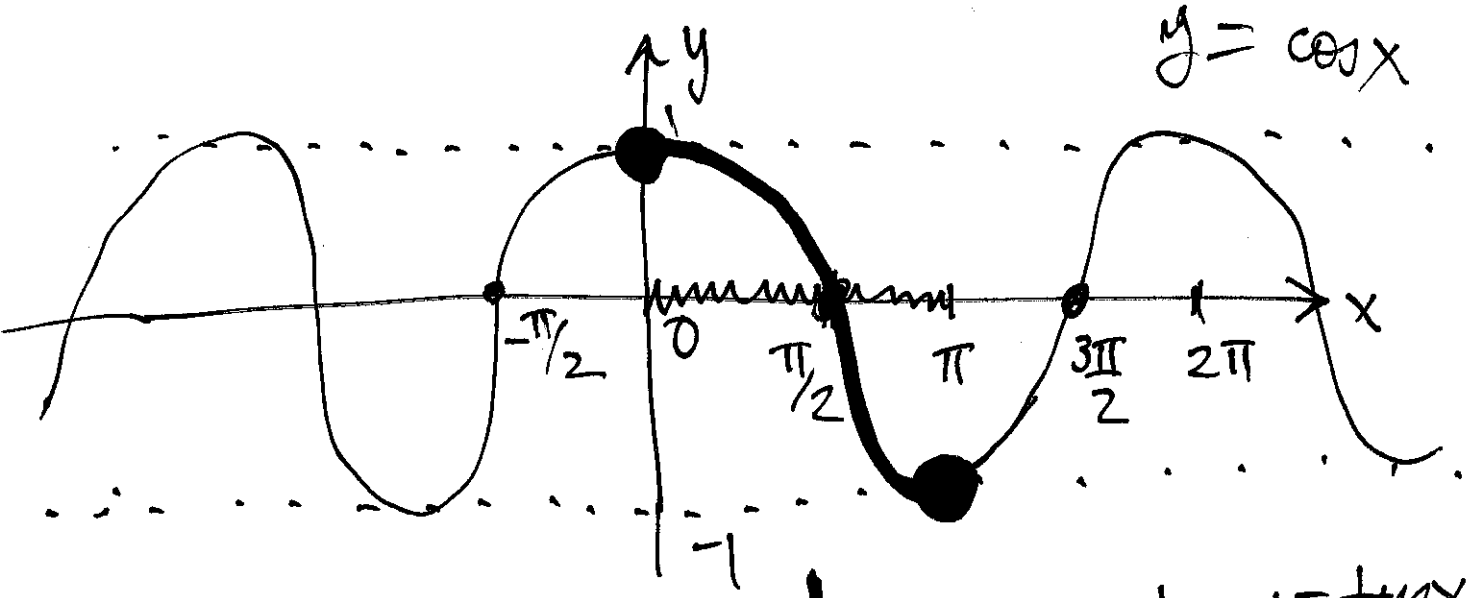
AND $x = -\frac{\pi}{3} + 2\pi m$, $m = 0, \pm 1, \pm 2 \ldots$

Graphs

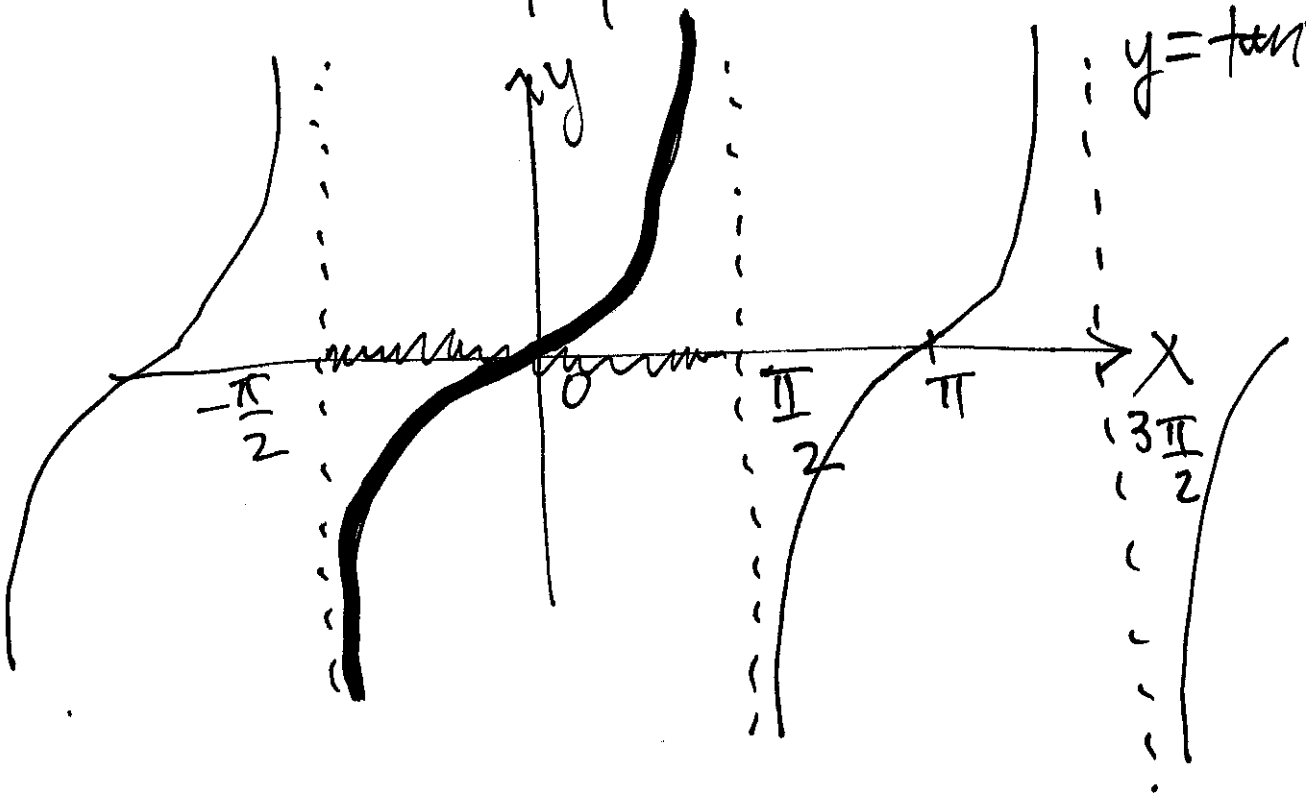
$y = \sin x$



$y = \cos x$

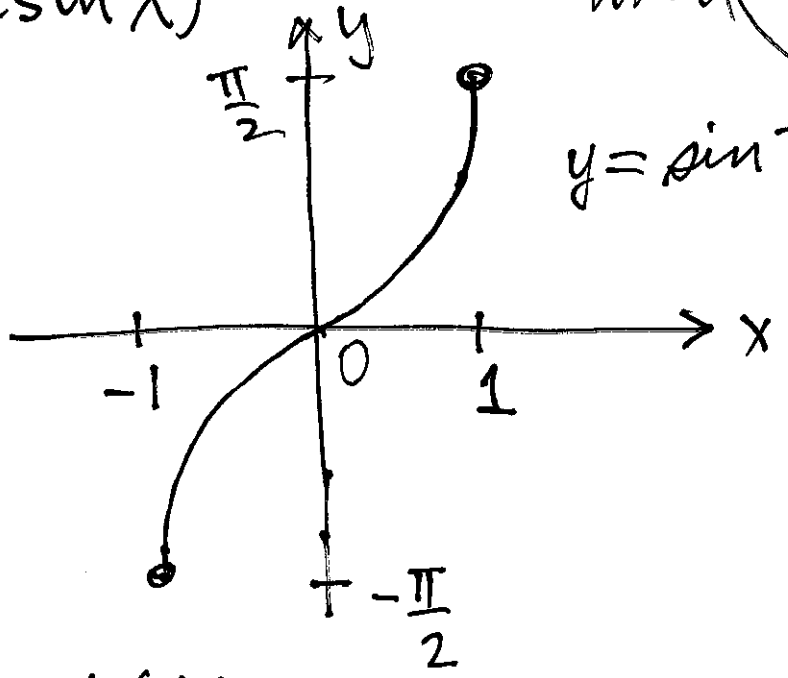


$y = \tan x$



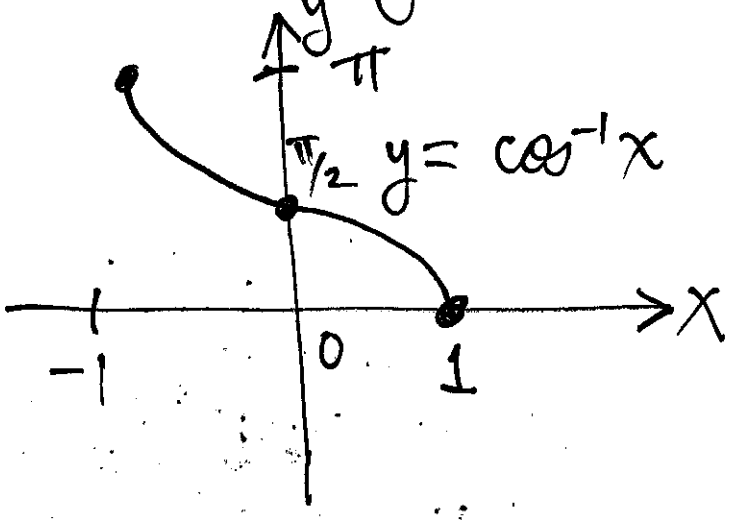
$y = \sin^{-1} x \iff \sin y = x$ when $\left(-\frac{\pi}{2} \le y \le \frac{\pi}{2}\right)$

$(y = \arcsin x)$



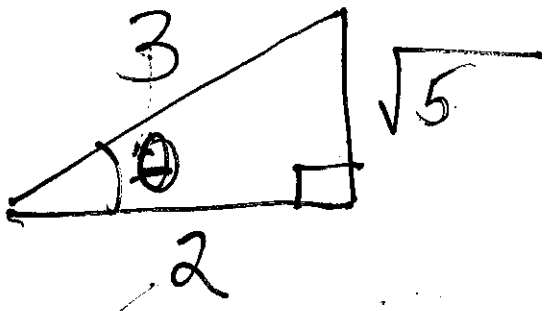
$y = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$ since $\sin\frac{\pi}{6} = \frac{1}{2}$ and $-\frac{\pi}{2} \le \frac{\pi}{6} \le \frac{\pi}{2}$

$y = \cos^{-1} x \iff \cos y = x$ when $(0 \le y \le \pi)$



If $\theta = \cos^{-1}\left(\frac{2}{3}\right)$, find $\sin\theta$.

i.e. $\cos\theta = \frac{2}{3}$



$\sin\theta = \frac{\sqrt{5}}{3}$ ✓