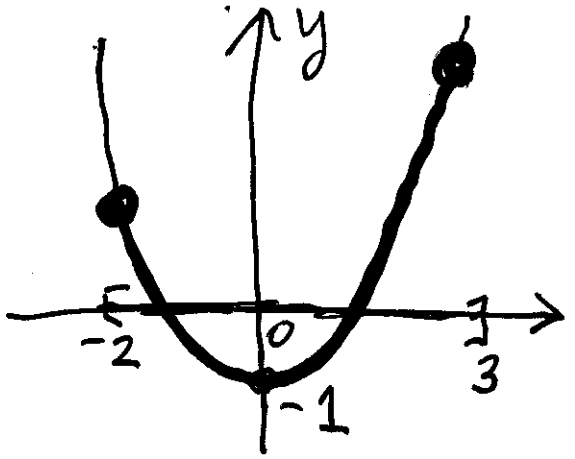
§4.1 – Maxima + Minima

Def: Let c be a point in a set I. Then

(i) $f(c)$ is an absolute max. value of f on I if $f(c) \geq f(x)$ for all x in I.

(ii) $f(c)$ is an absolute min. value of f on I if $f(c) \leq f(x)$ for all x in I.

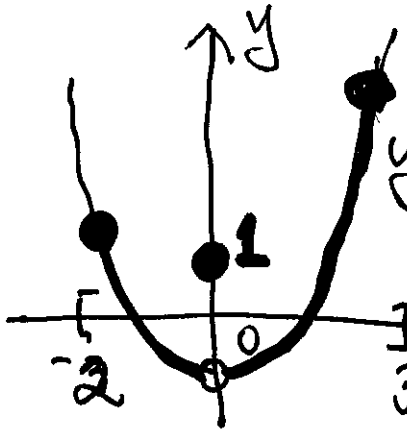
For eg. $y = f(x) = x^2 - 1$ and $I = [-2, 3]$



abs max value is $f(3) = 8$

abs. min value is $f(0) = -1$

let $g(x) = \begin{cases} x^2 - 1, & x \neq 0 \\ 1, & x = 0 \end{cases}$



$y = g(x)$

No abs min value!

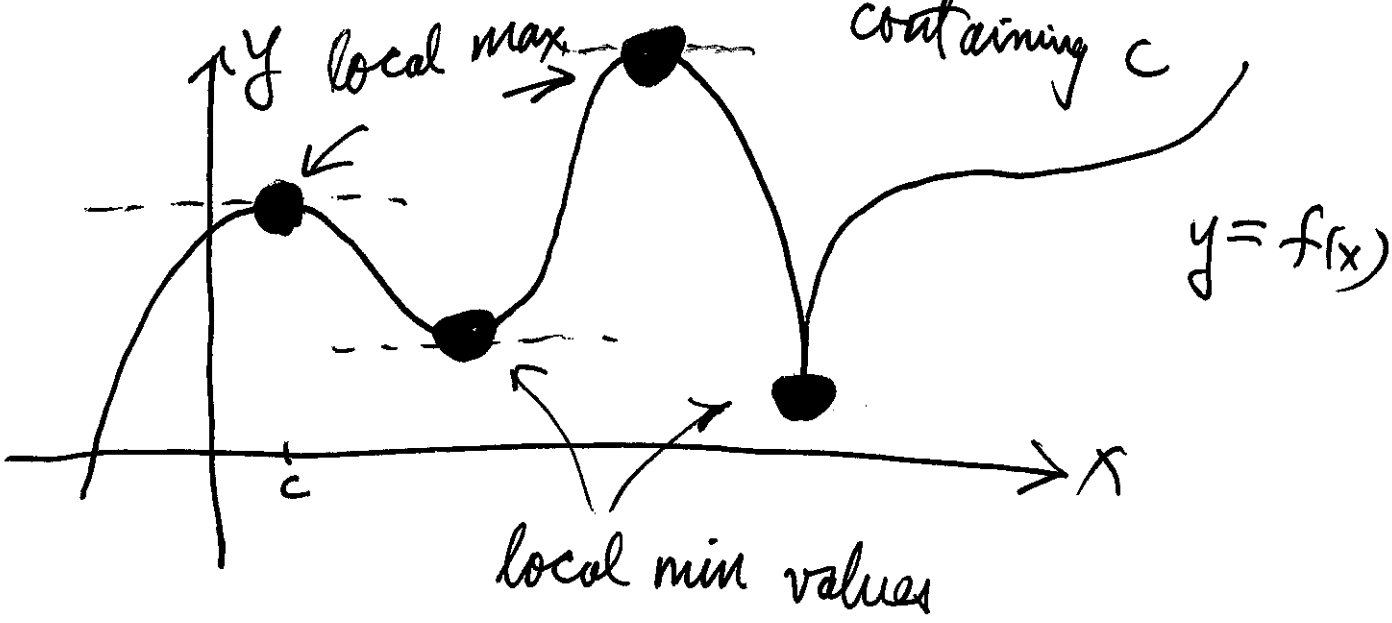
Absolute Extreme Value Thm:

If f is cont. on $[a, b] \Rightarrow$ f always has an abs. max value and abs. min. value

Def: If c is an interior point of interval I where f is defined

(i) f(c) is a local/relative max. value of f if $f(c) \geq f(x)$ for all x in some small interval containing c

(ii) f(c) is local/relative min. value of f if $f(c) \leq f(x)$ for all x in small interval containing c



Local Extreme Value Thm

If f has a local max/min value at c
$\Rightarrow$ either $f'(c)=0$ or $f'(c)$ DNE.

Def: An interior pt c in domain of f is a **critical point** of f if either $f'(c)=0$ or $f'(c)$ DNE

Ex1 Find all critical pts

(a) $f(x)=x^2(x+5)^{1/3}$

$$f'(x)=x^2\left\{\frac{1}{3}(x+5)^{-2/3}\right\}+(x+5)^{1/3}\{2x\}$$

$$=(x+5)^{-2/3}\left[\frac{x^2}{3}+(x+5)(2x)\right]$$

$$\vdots$$

$$=\frac{x(7x+30)}{3(x+5)^{2/3}}$$

Critical pts:
$x=0$
$x=-30/7$
$x=-5$

Finding Abs. Extrema Method

$f(x)$ must be cont. on $[a,b]$

1. Find all admissible critical pts in (a,b)
2. Make a table of values of $f(x)$ at critical pts and endpts of interval.
3. Choose largest & smallest values

Ex2 Find abs. extrema

(a) $f(x)=\frac{3}{4}x^4-x^3-9x^2+1$; $I=[-3,1]$

$\Rightarrow f'(x)=3x(x-3)(x+2)=0 \Rightarrow x=0$

not admissible ← $x=3$

$x=-2$

x	$f(x)=\frac{3}{4}x^4-x^3-9x^2+1$	
0	1	
−2	−15	← abs. min value
−3	7.75	← abs. max value
1	−8.25	

Note: $f(3)=-46.25$ ~~3 not in $[-3,1]$~~

(b) $f(x) = \sin^{-1}x - x^2$; $I = [0, 1]$

$$f'(x) = \frac{1}{\sqrt{1-x^2}} - 2x = 0$$

$$\frac{1}{\sqrt{1-x^2}} = 2x$$

$$\frac{1}{1-x^2} = 4x^2$$

$$\Rightarrow \quad 4x^4 - 4x^2 + 1 = 0$$

$$(2x^2 - 1)^2 = 0$$

$$2x^2 = 1$$

$$x = \pm \frac{1}{\sqrt{2}}$$

(b) $f(x) = \sin^{-1}x - x^2$ $\quad [0,1]$

$f'(x) = \frac{1}{\sqrt{1-x^2}} - 2x$ continue...