Lesson 20

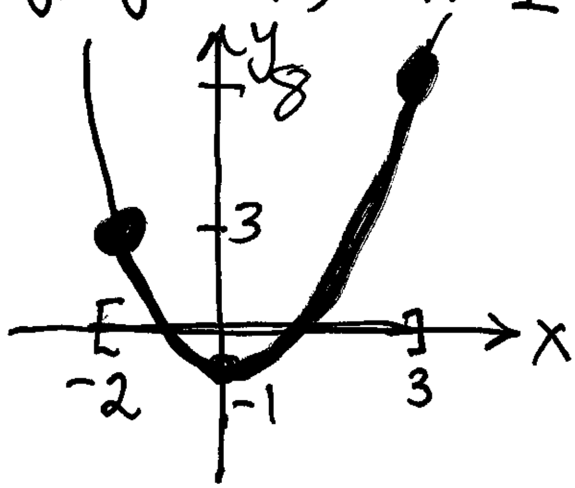
§4.1 – Maxima & Minima

Def: Let c be a point in a set I. Then

(i) $f(c)$ is an absolute max. value of f on I if $f(c) \geq f(x)$ for all x in I

(ii) $f(c)$ is an absolute min. value of f on I if $f(c) \leq f(x)$ for all x in I.

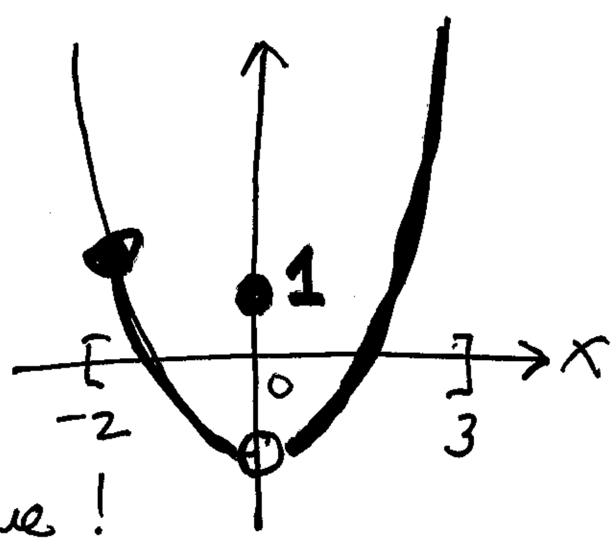
For eg, $y = f(x) = x^2 - 1$ and $I = [-2, 3]$



abs max value is $f(3) = 8$

abs min value is $f(0) = -1$

let $g(x) = \begin{cases} x^2 - 1, & \text{if } x \neq 0 \\ 1, & \text{if } x = 0 \end{cases}$



$g(x)$ has no abs. min value!

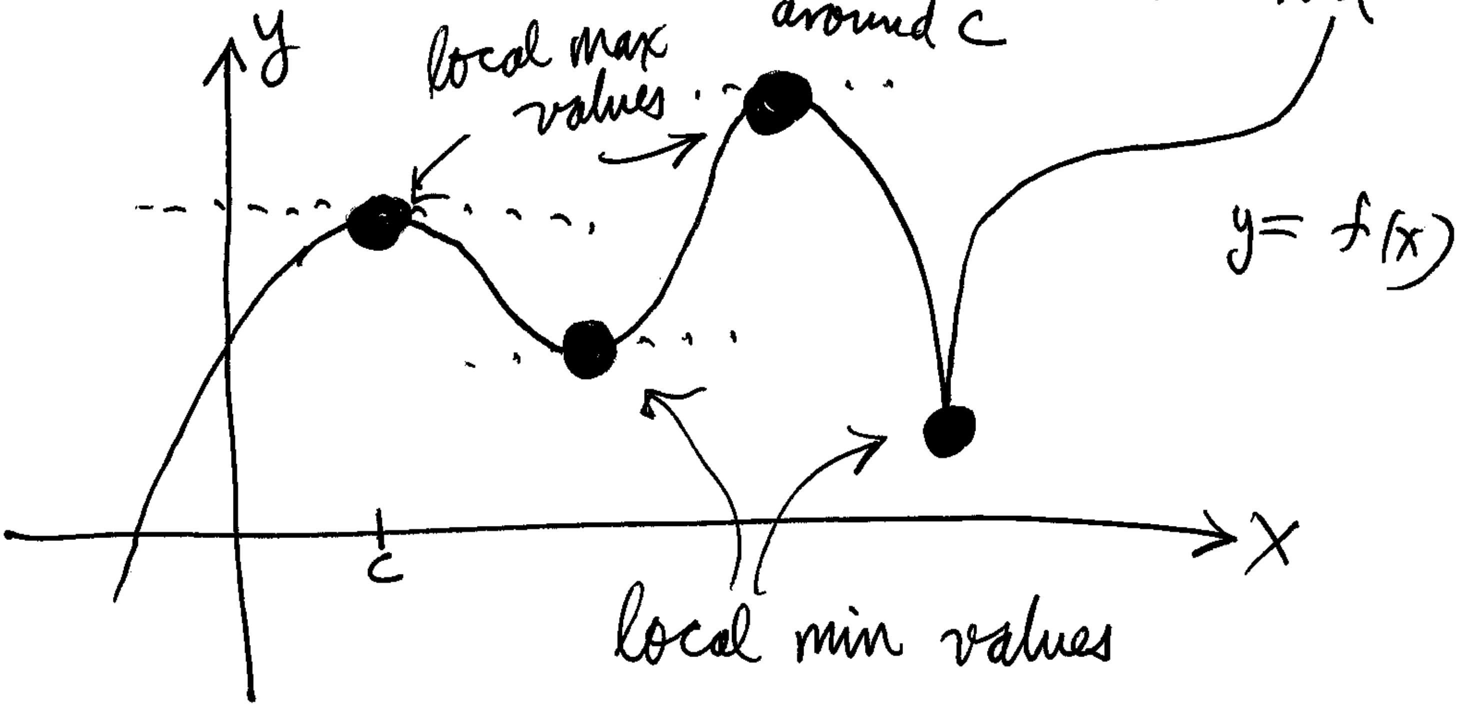
Absolute Extreme Value Thm

If f is cont. on $[a,b] \Rightarrow$ f always has an abs. max value & abs. min value.

Def: If c is an interior point of interval I where f is defined then

(i) $f(c)$ is a local/relative max value of f if $f(c) \geq f(x)$ for all x in some small interval around c

(ii) $f(c)$ is local/relative min value of f if $f(c) \leq f(x)$ for all x in some small interval around c



Local Extreme Value Thm

If f has local max/min value at c
$\Rightarrow$ either $f'(c)=0$ or $f'(c)$ DNE

Def: An interior point c in domain of f is a critical point of f if either $f'(c)=0$ or $f'(c)$ DNE.

$\boxed{\text{Ex1}}$ Find all critical pts

$$f(x)=x^2(x+5)^{1/3}$$

$$f'(x)=x^2\left\{\frac{1}{3}(x+5)^{-2/3}\right\}+(x+5)^{1/3}\{2x\}$$

$$=(x+5)^{-2/3}\left[\frac{x^2}{3}+(x+5)(2x)\right]$$

$$f'(x)=\frac{x(7x+30)}{3(x+5)^{2/3}}=0 \Rightarrow \begin{array}{l} x=0 \\ x=-\frac{30}{7} \\ x=-5 \end{array}$$

Finding Abs. Extrema Method

$f(x)$ must be cont. on $[a, b]$

1. Find all admissible critical pts in (a, b)
2. Make a table of values of $f(x)$ at critical pts and at endpoints $x = a$, $x = b$
3. Choose largest & smallest values

Ex 2 Find abs. extrema:

(a) $f(x) = \frac{3}{4}x^4 - x^3 - 9x^2 + 1$; $I = [-3, 1]$

⋮

$\Rightarrow f'(x) = 3x(x-3)(x+2) = 0$ $\quad x = 0$

not admissible $\quad x = 3$

$x = -2$

critical pts

x	$f(x) = \frac{3}{4}x^4 - x^3 - 9x^2 + 1$
0	1
−2	−15 ← abs min value
−3	7.75 ← abs max value
1	−8.25

Note: $\cancel{f(3) = -46.25}$