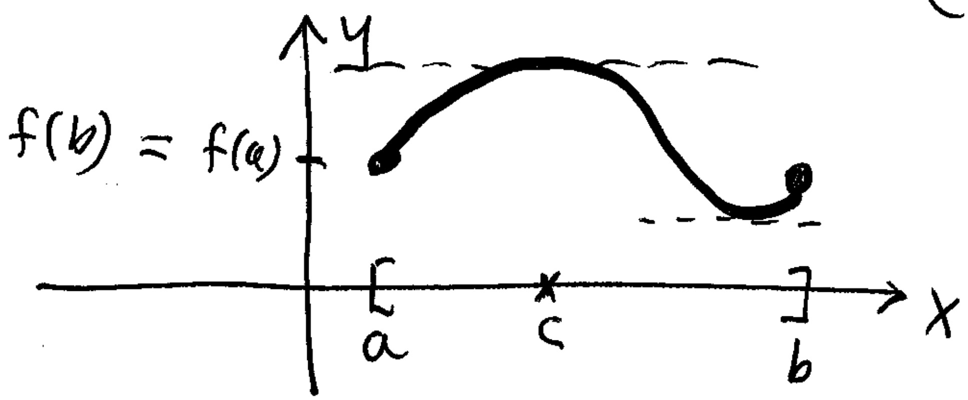
Lesson 21

§§4.2+4.3 ~~Derivatives~~

Rolle's Thm: If f is cont. on $[a,b]$

~~f diff.~~ on (a,b) and $f(a) = f(b)$

then $f'(c) = 0$ for some c in (a,b).



For eg $f(x) = x^3 - 2x^2 - 8x + 2$; $[-2, 4]$

satisfies Rolle's Thm

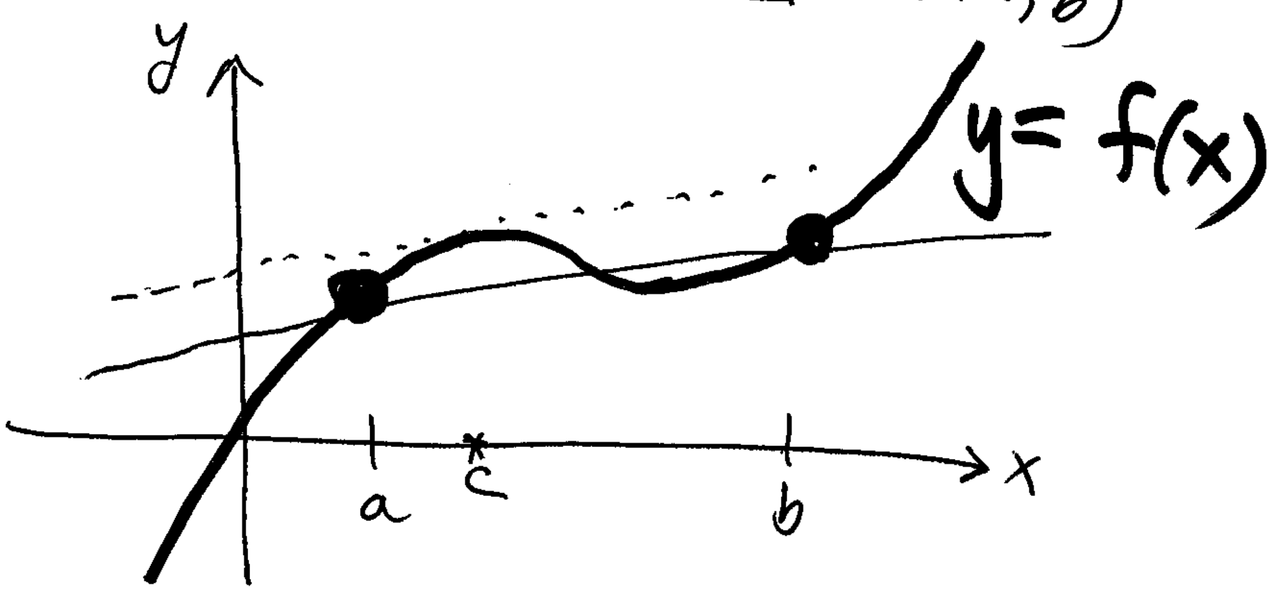
$$f(-2) = 2 = f(4)$$

$\therefore$ $f'(c) = 0$ for some $-2 < c < 4$

$3c^2 - 4c - 8 = 0$ $\quad c = \dfrac{4 \pm \sqrt{112}}{6}$ ✓

Mean Value Thm: If f is cont on $[a, b]$, diff. on (a, b) then

$$\boxed{\frac{f(b) - f(a)}{b - a} = f'(c)}$$ for some c in (a, b)



$\boxed{\text{Ex1}}$ Spring 2019 Final #17

If $f(x) = x^3 - 2x^2 - 3x$, $0 \leq x \leq 2$

find c satisfying MVT

A. $3/4$ B. 1 C. $5/4$ D. $4/3$ E. $3/2$

Soln: $\dfrac{f(2) - f(0)}{2 - 0} = f'(c)$

$$-3 = 3c^2 - 4c - 3$$

$$\vdots$$

$$\Rightarrow \quad c = 0 \quad \text{or} \quad c = 4/3$$

Recall $0 < c < 2$

Fact: If $f'(x) > 0$ on $I \Rightarrow f$ is increasing on I

If $f'(x) < 0$ on $I \Rightarrow f$ is decreasing on I

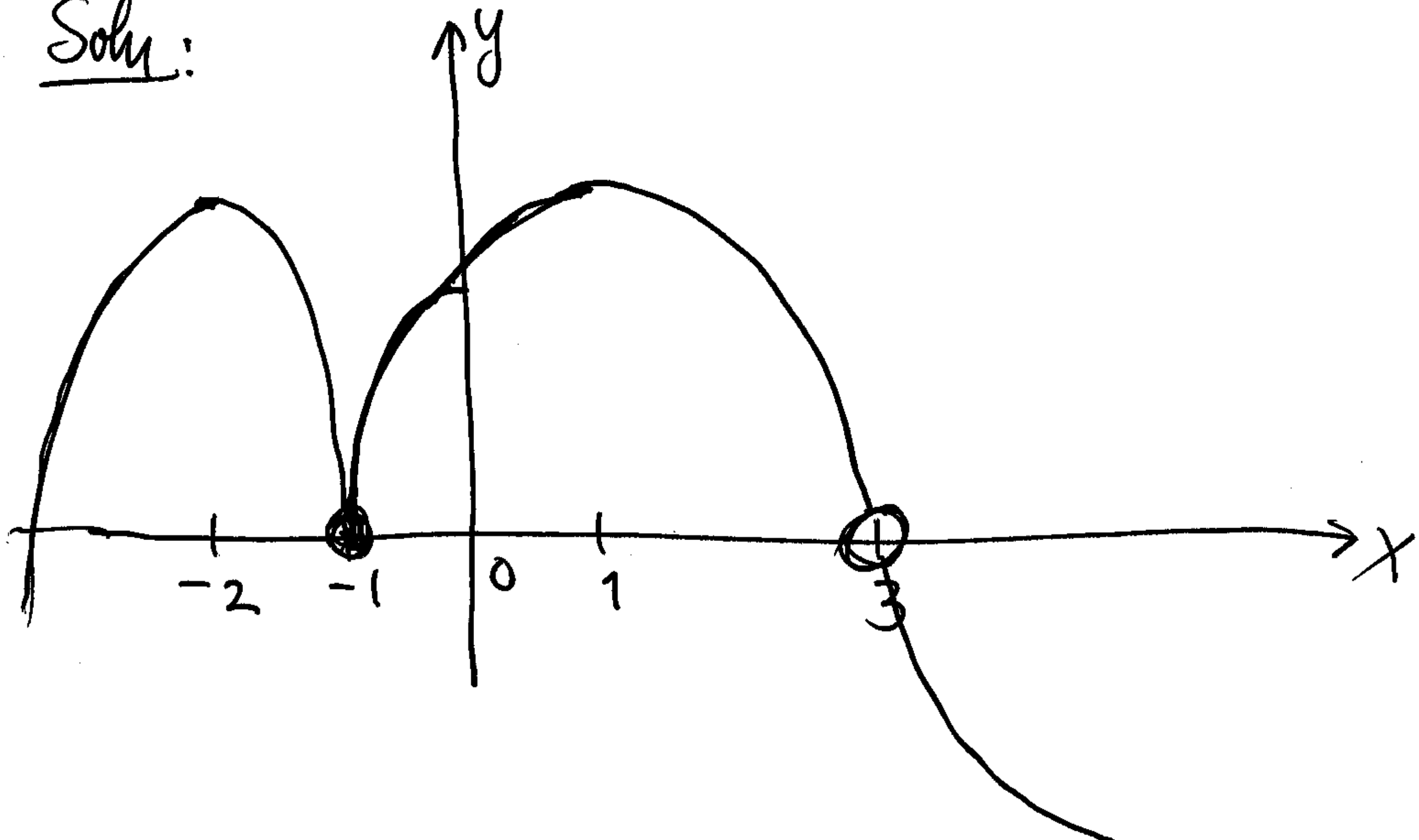
$\boxed{\text{Ex 2}}$ Sketch graph of $f(x)$

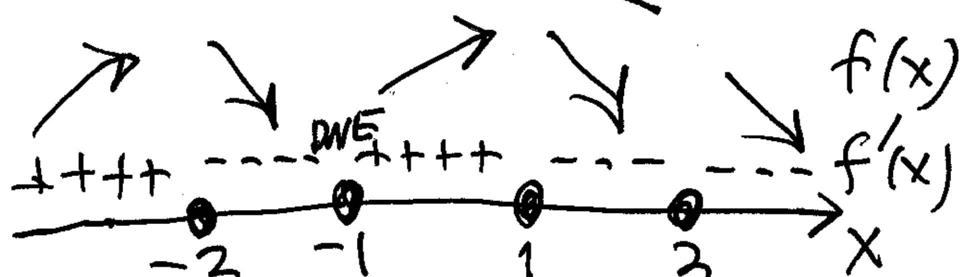
$f' > 0$ on $(-\infty, -2) \cup (-1, 1)$

$f' < 0$ on $(-2, -1) \cup (1, 3) \cup (3, \infty)$

$f(-1) = 0$; $f'(-1)$ DNE; $f(3)$ DNE

Soln:

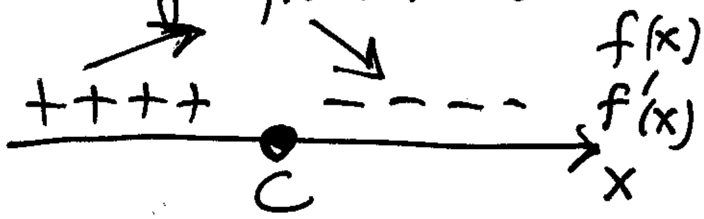




First Derivative Test: Suppose f cont on I containing critical pt c, f diff on I (except possibly at c), then

① If f' changes from $+$ to $-$

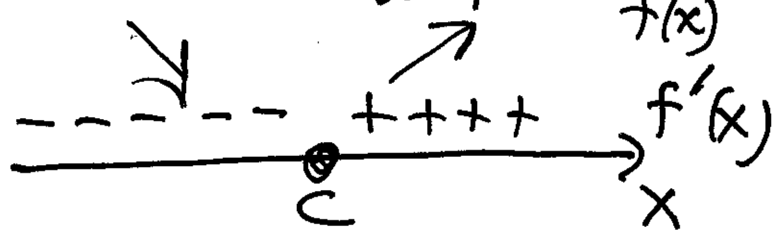
i.e.



$\Rightarrow f(c)$ is a local max value

② If f' changes from $-$ to $+$

i.e.

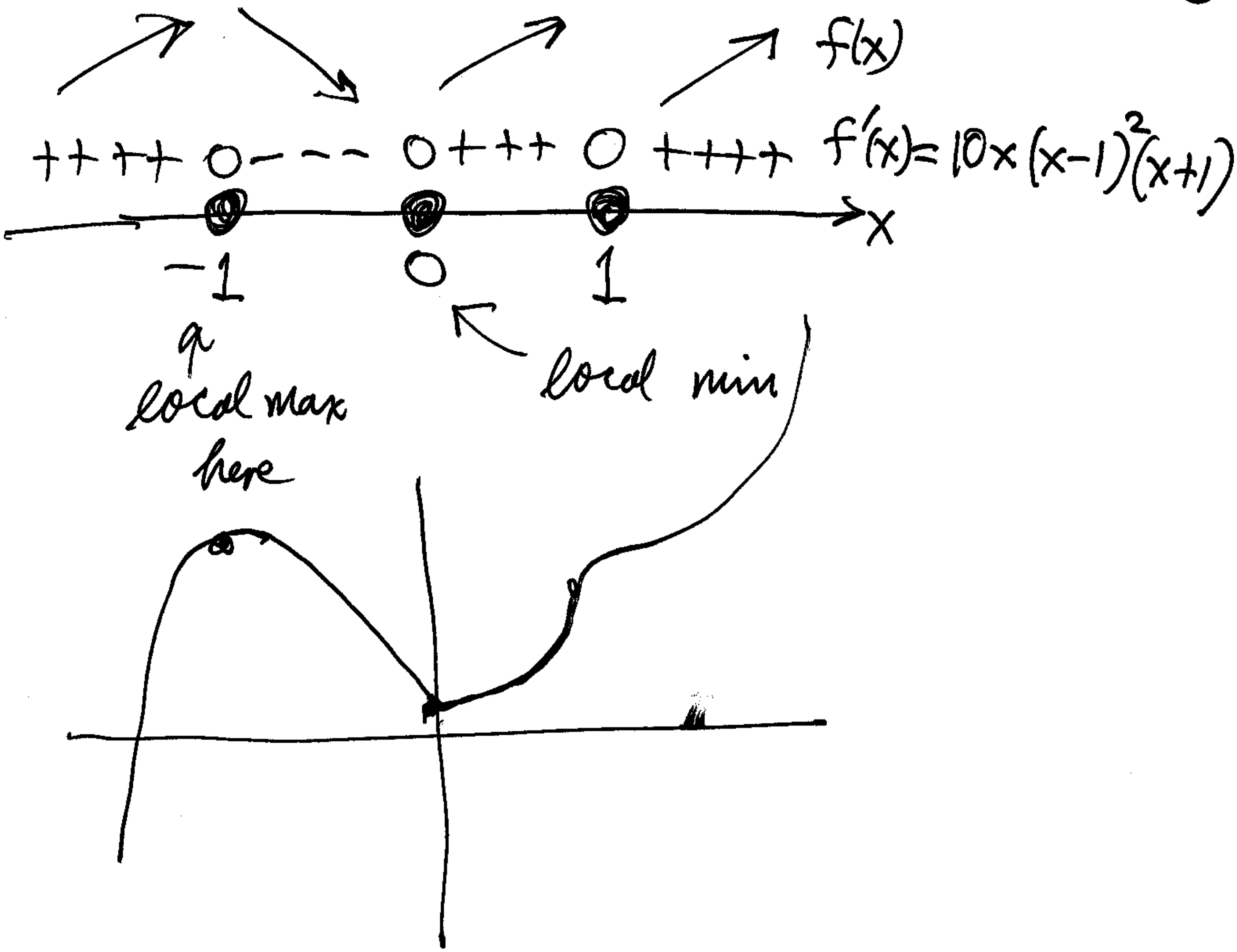


$\Rightarrow f(c)$ is a local min value.

$\boxed{\text{Ex3}}$ Find local extrema

ⓐ $\underline{f(x) = 2x^5 - \frac{5}{2}x^4 - \frac{10}{3}x^3 + 5x^2 + 42}$

$\Rightarrow f'(x) = 10x(x-1)^2(x+1)$


f(x)
+++++ 0 - - - 0 ++++ 0 ++++
$f'(x) = 10x(x-1)^2(x+1)$
x
-1
0
1
a local max here
local min