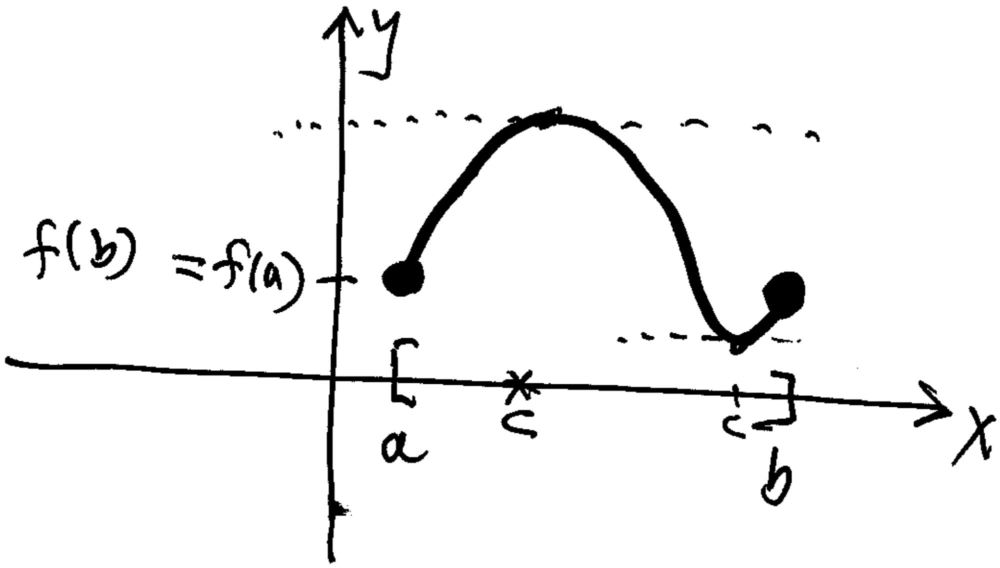
Lesson 21

§§4.2 + 4.3 – Derivatives.

Rolle's Thm: If f is cont. $[a, b]$ diff. on (a, b) and $f(a) = f(b)$ then $f'(c) = 0$ for some c in (a, b)



For eg $f(x) = x^3 - 2x^2 - 8x + 2$; $[-2, 4]$

satisfies Rolle's Thm : $f(-2) = f(4)$

$\therefore f'(c) = 0$ for some c in $(-2, 4)$

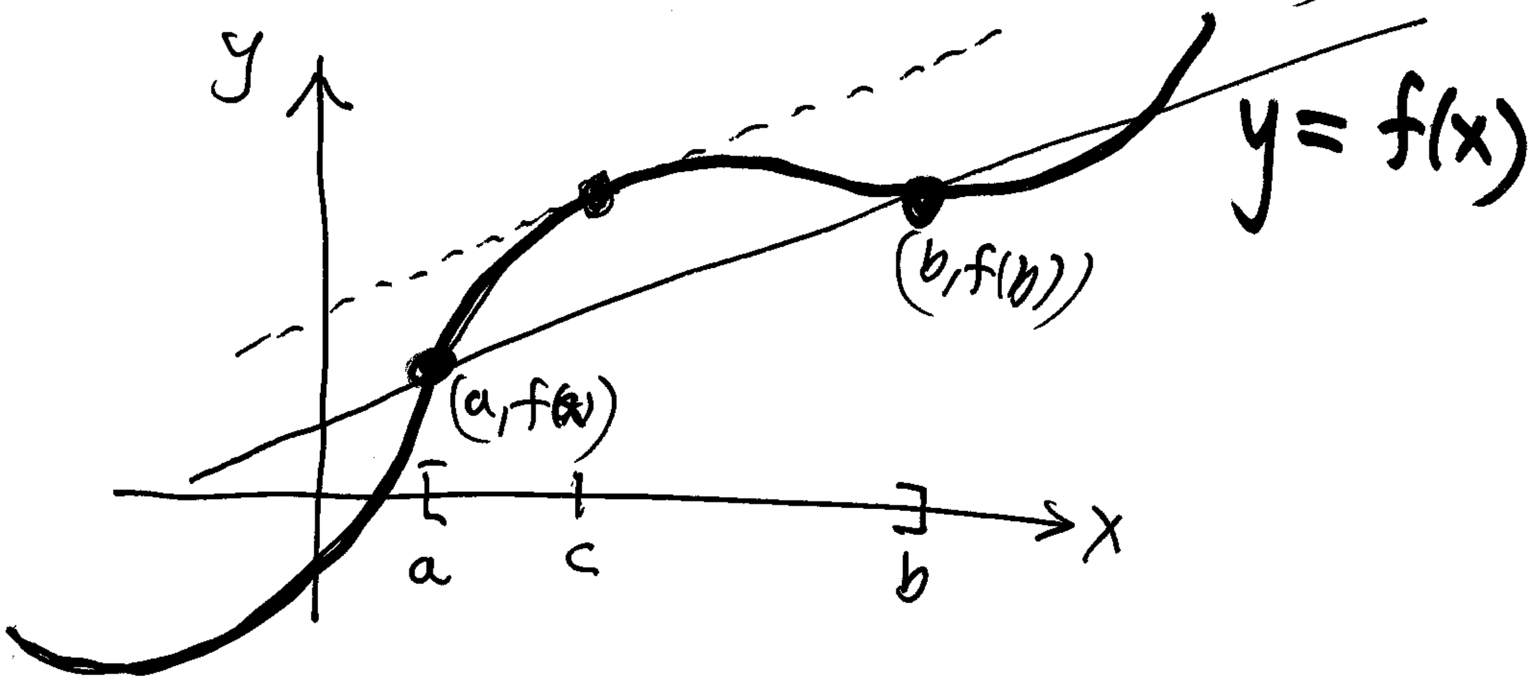
$3c^2 - 4c - 8 = 0$

$$\Rightarrow c = \frac{4 \pm \sqrt{112}}{6}$$

Mean Value Thm (MVT)

If f is cont. on $[a,b]$ and diff on (a,b),

then $\boxed{\dfrac{f(b)-f(a)}{b-a} = f'(c)}$ for some c in (a,b)



$\boxed{\text{Ex1}}$ Spring 2019 Final #17

If $f(x) = x^3 - 2x^2 - 3x$, $0 \le x \le 2$,

find c satisfying MVT

A. $3/4$ B. 1 C. $5/4$ D. $4/3$ E. $3/2$

Soln: $\dfrac{f(2)-f(0)}{2-0} = f'(c)$

$$-3 = 3c^2 - 4c - 3$$

Solve for c, algebra $\Rightarrow c = 0$ or

since $0 < c < 2 \rightarrow$ $c = 4/3$

Fact: If $f'(x) > 0$ on $I \Rightarrow f$ increasing on I

If $f'(x) < 0$ on $I \Rightarrow f$ decreasing on I.

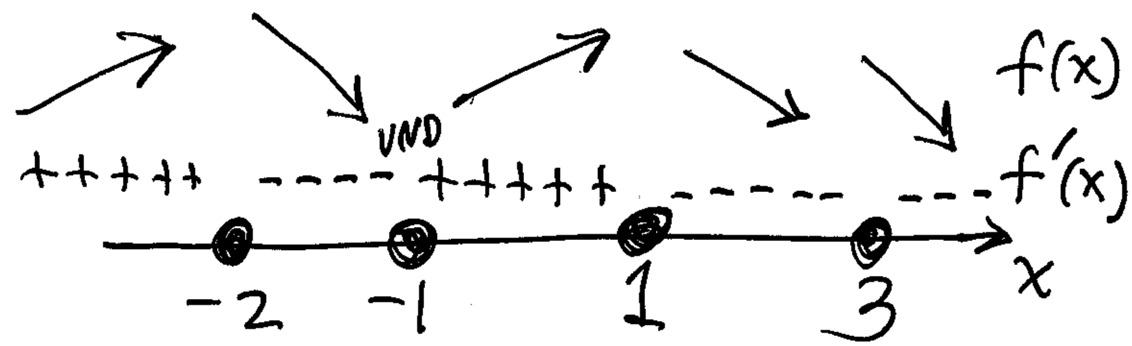
$\boxed{\text{Ex 2}}$ Sketch graph of $f(x)$:

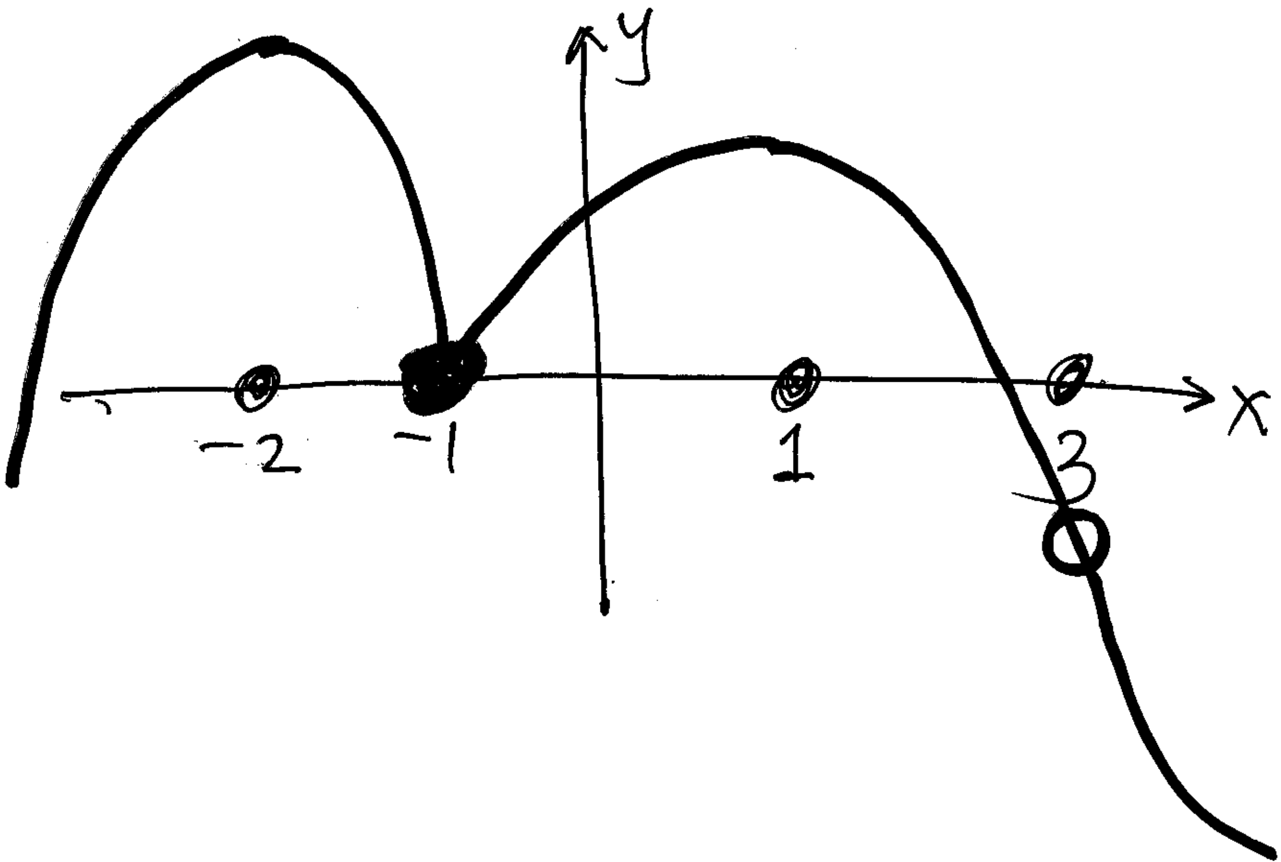
$f' > 0$ on $(-\infty, -2) \cup (-1, 1)$

$f' < 0$ on $(-2, -1) \cup (1, 3) \cup (3, \infty)$

$f(-1) = 0$, $f'(-1)$ DNE ; $f(3)$ DNE

Soln:





First Derivative Test – Suppose f is cont. on I containing critical pt c, f diff. on I (except possibly at c), then

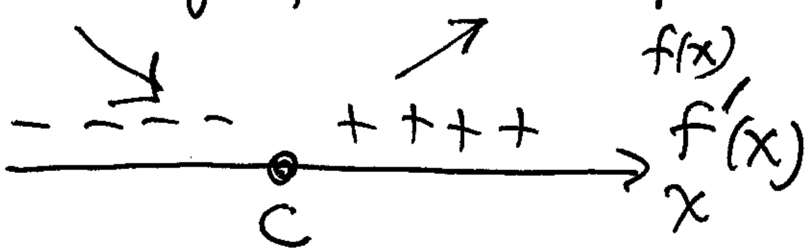
(1) If f' changes sign from $+$ to $-$

i.e.

$\Rightarrow f(c)$ is a local max value

(2) If f' changes from $-$ to $+$

i.e.



$\Rightarrow f(c)$ is a local min value

$\boxed{\text{Ex3}}$ Find local extrema

$$f(x) = 2x^5 - \frac{5}{2}x^4 - \frac{10}{3}x^3 + 5x^2 + 42$$

Soln: $\Rightarrow f'(x) = 10x(x-1)^2(x+1)$