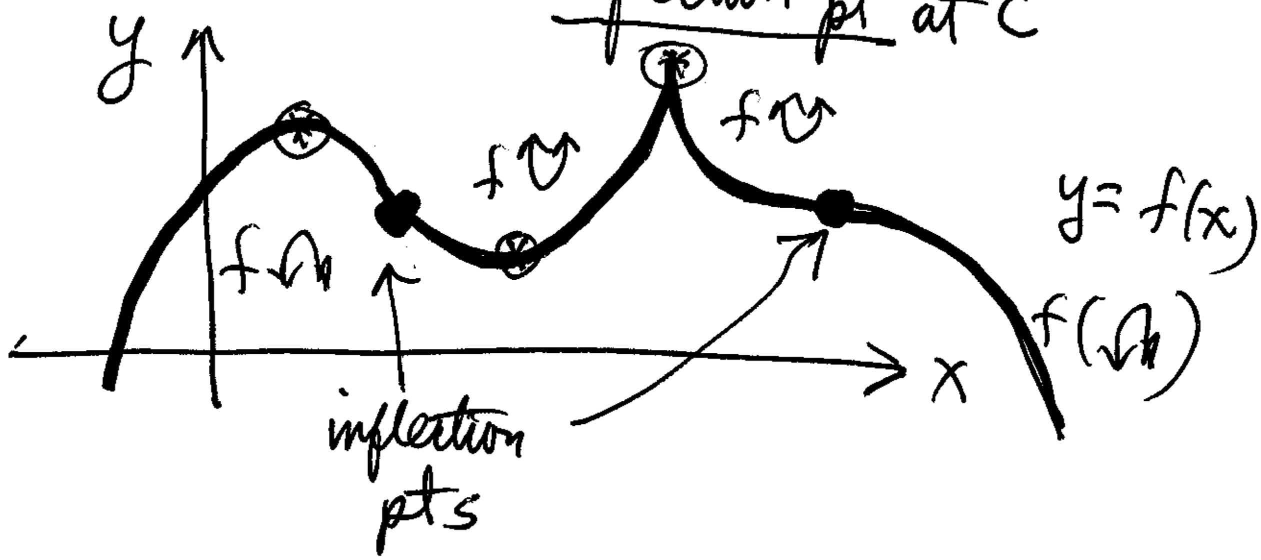
§4.3 – Concavity + 2nd Derivative Test

Def: Let f be diff on open interval I.

(i) If f′ is increasing on I ⇒ f is concave up on I (f ↻)

(ii) If f′ is decreasing ⇒ f is concave down on I (f ↷)

(iii) If f is cont. at c and changes concavity at c ⇒ f has inflection pt at c



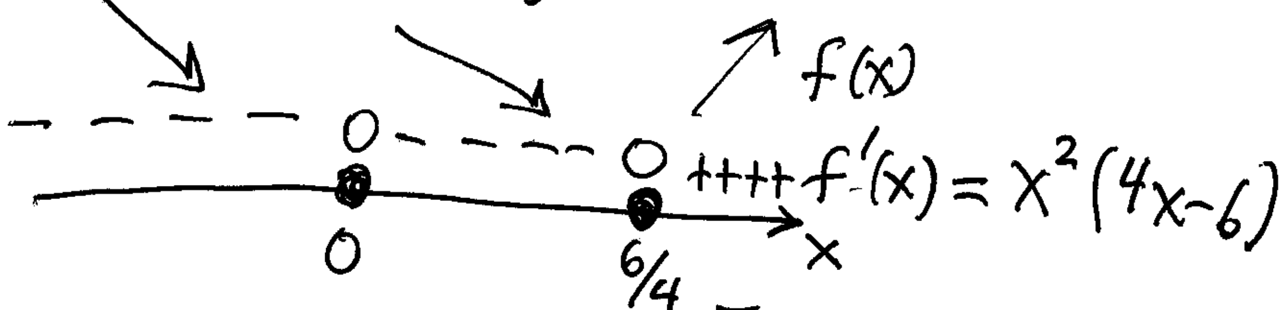
Concavity Test: Suppose f'' cont. on open interval I

1. If $f''(x) > 0$ on $I \Rightarrow f$
2. If $f''(x) < 0$ on $I \Rightarrow f$
3. If c is pt in I where f'' changes sign $\Rightarrow f$ has inflection pt at c

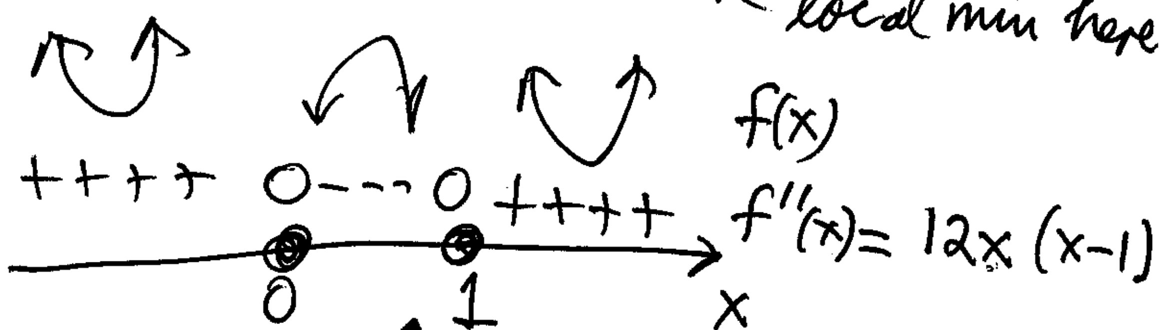
For eg, $f(x) = x^4 - 2x^3 + 1$

$$\Rightarrow f'(x) = 4x^3 - 6x^2 = x^2(4x-6)$$

$$f''(x) = 12x(x-1)$$



local min here!



inflection pts

Second Derivative Test: Suppose f'' cont on open interval I, contains c, suppose $\boxed{f'(c)=0}$

Then

1. If $f''(c)>0 \Rightarrow f$ has local min at c
2. If $f''(c)<0 \Rightarrow f$ has local max at c
3. If $f''(c)=0 \Rightarrow$ test is inconclusive (use 1st Deriv. Test!)

$\boxed{\text{Ex 1}}$ Find local extrema

(a) $f(x)=3x^4-4x^3-6x^2+12x+5$

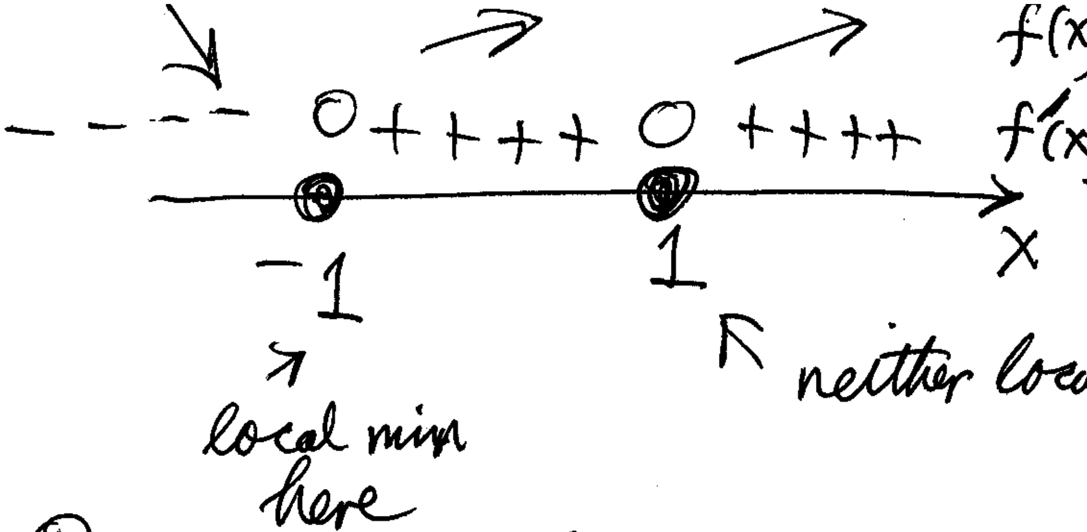
$\Rightarrow f'(x)=12(x-1)^2(x+1)$ ✓

$f''(x)=12(x-1)(3x+1)$ ✓✓

Critical pts are $x=1$ and $x=-1$

For $x=-1$: $f''(-1)=48>0 \Rightarrow 2^{nd}$ D. Test says $f(-1)$ is local min value

For $x=1$: $f''(1)=0$ 2^{nd} D. Test no good.



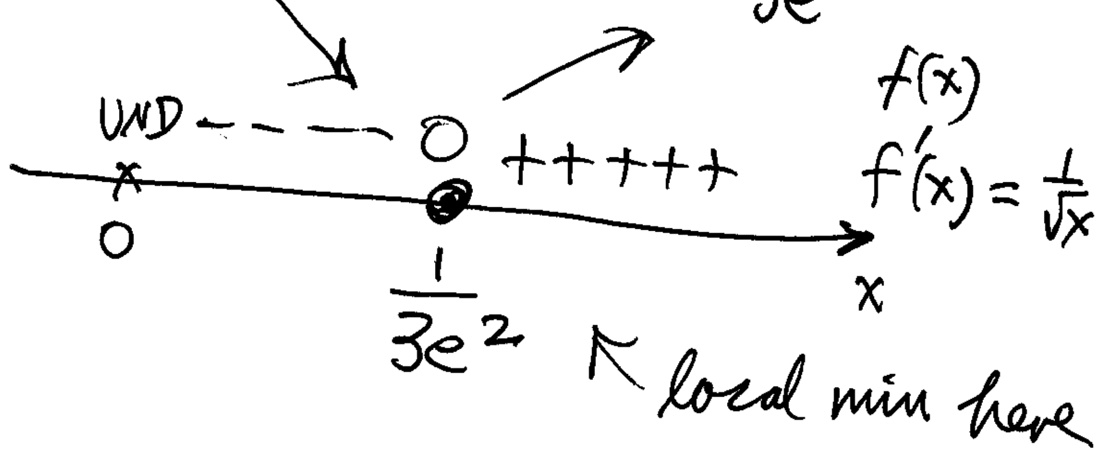
$f'(x) = 12(x-1)^2(x+1)$

(b) $f(x) = \sqrt{x}\,\ln(3x)$

$$\Rightarrow f'(x) = \frac{1}{x^{1/2}}\Big[\underbrace{1 + \tfrac{1}{2}\ln(3x)}_{=0}\Big]$$

$$1 + \tfrac{1}{2}\ln(3x) = 0 \Rightarrow e^{\ln(3x)} = e^{-2}$$

$$\Rightarrow 3x = e^{-2} \Rightarrow x = \frac{1}{3e^2}$$



$f'(x) = \frac{1}{\sqrt{x}}\Big[1 + \tfrac{1}{2}\ln(3x)\Big]$