Lesson 22

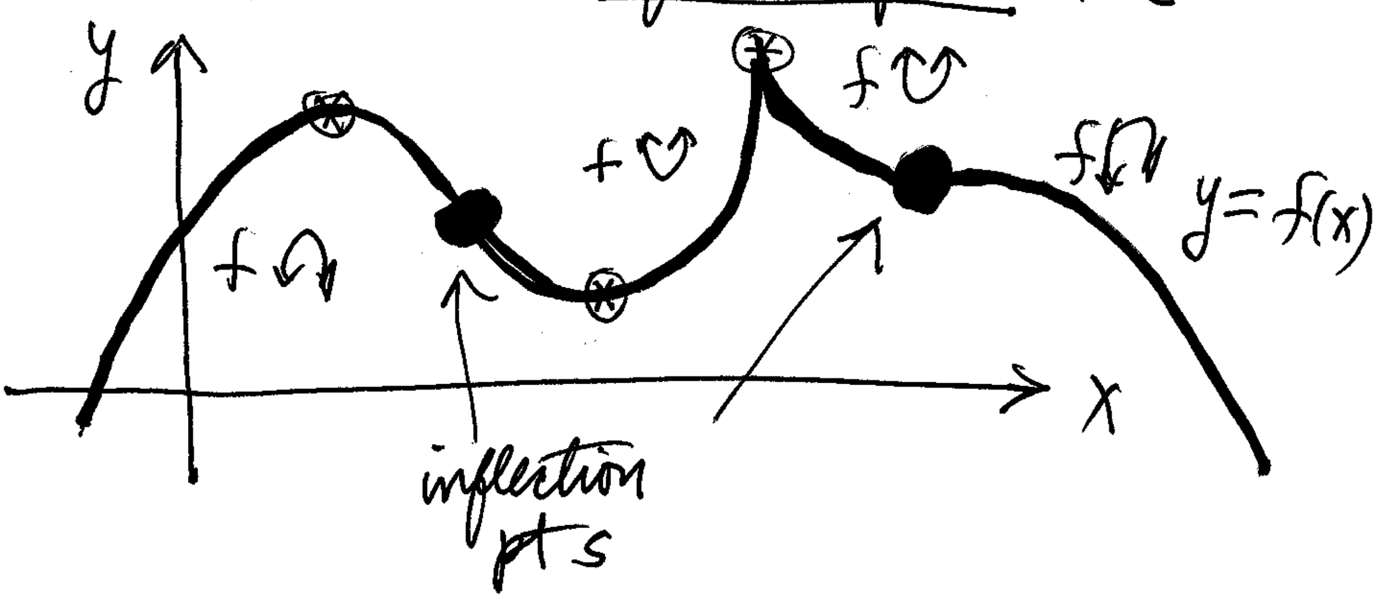
§4.3 – Concavity + 2nd Derivative Test

Def: Let f be diff on open interval I

(i) If f' is increasing on $I \Rightarrow f$ is concave up on I
f ↻

(ii) If f' is decreasing $\Rightarrow f$ is concave down on I
f ↷

(iii) If f is cont. at c and concavity changes at $c \Rightarrow f$ has inflection point at c



Concavity Test: Suppose f'' cont on open interval I

① If $f''(x) > 0$ on $I \Rightarrow f$ $\cup$ on I

② If $f''(x) < 0$ on $I \Rightarrow f$ $\cap$ on I

③ If c is a pt in I where f'' changes sign
$\Rightarrow f$ has inflection pt at c

For eg, $f(x) = x^4 - 2x^3 + 1$

$f'(x) = x^2(4x-6)$

$f''(x) = 12x(x-1)$

$f(x)$: $\searrow$ $\quad$ $\searrow$ $\quad$ $\nearrow$

$f'(x) = x^2(4x-6)$: $- - - -$ 0 $- - - -$ 0 $++++$

x: 0 $\quad$ $6/4$ ← local min here

$f(x)$: $\cup$ $\quad$ $\cap$ $\quad$ $\cup$

$f''(x) = 12x(x-1)$: $++++$ 0 $- - -$ 0 $++++$

x: 0 $\quad$ 1

inflection pts

Second Derivative Test: Suppose f'' cont. on open interval I, contains c, and suppose $\boxed{f'(c)=0}$ Then

① If $f''(c)>0 \Rightarrow f$ has local min at c

② If $f''(c)<0 \Rightarrow f$ has local max at c

③ If $f''(c)=0 \Rightarrow$ test inconclusive!
(use 1^{st} Derivative Test)

$\boxed{\text{Ex 1}}$ Find local extrema

(a) $f(x)=3x^4-4x^3-6x^2+12x+5$

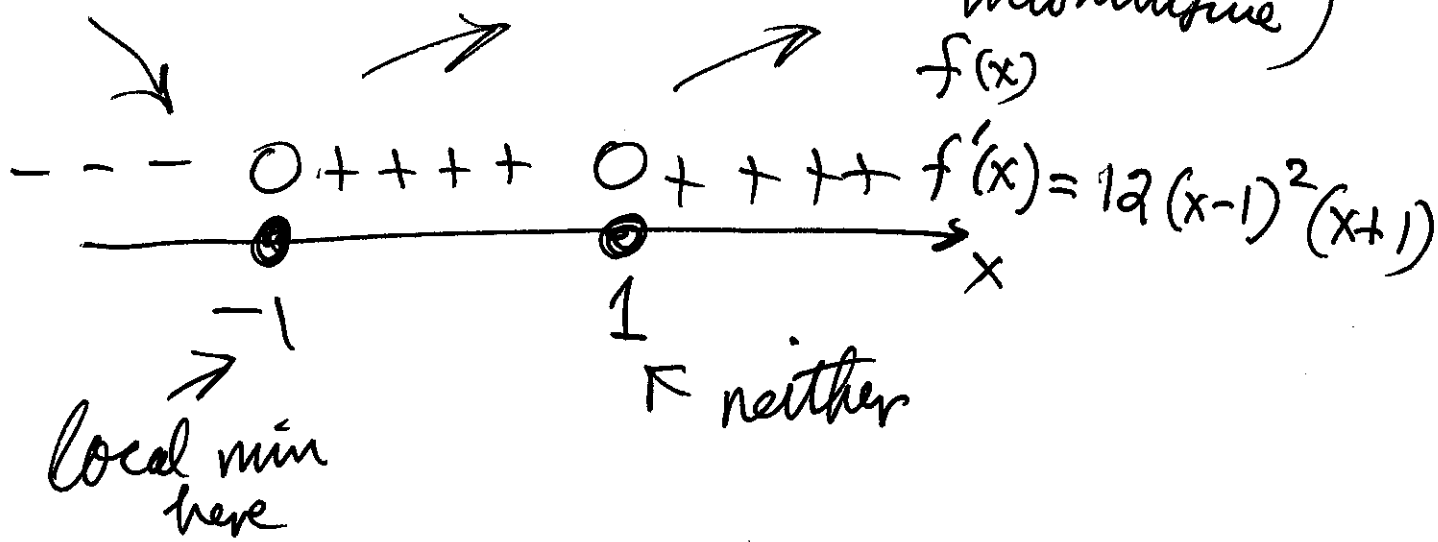
$\Rightarrow f'(x)=12(x-1)^2(x+1)$ ✓

$f''(x)=12(x-1)(3x+1)$ ✓✓

$\Rightarrow$ critical pts are $x=1$, $x=-1$

For $x=-1$: $f''(-1)=48>0 \Rightarrow f(-1)$ is local min value by 2^{nd} D. Test

For $x=1$: $f''(1)=0$ (test fails, inconclusive)



ⓑ $\underbrace{f(x)=\sqrt{x}\,\ln(3x)}$

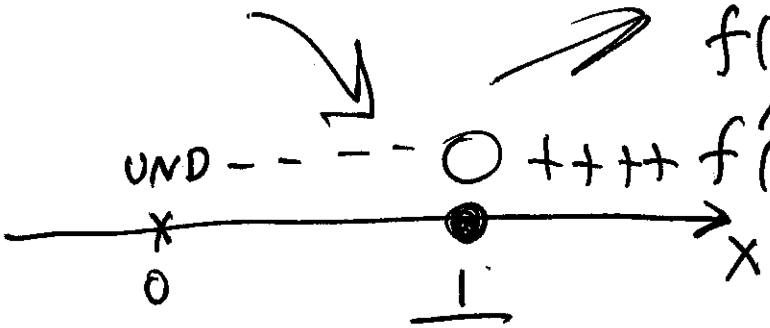
$\Rightarrow f'(x)=\frac{1}{\sqrt{x}}\left[\underbrace{1+\frac{1}{2}\ln(3x)}_{=0}\right]$

$1+\frac{1}{2}\ln(3x)=0 \Rightarrow \ln(3x)=-2$

$e^{\ln(3x)}=e^{-2}$

$3x=e^{-2}$

$x=\frac{1}{3e^2}$



$$f'(x) = \frac{1}{\sqrt{x}}\left[1 + \frac{1}{2}\ln(3x)\right]$$

$\frac{1}{3e^2}$ ← local min here!