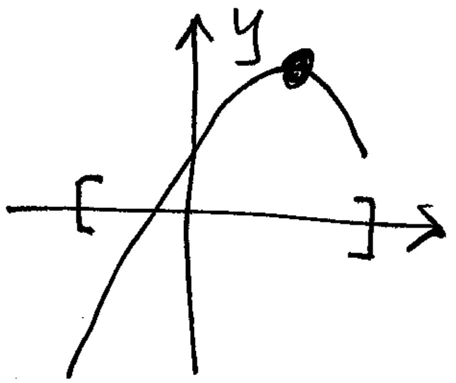
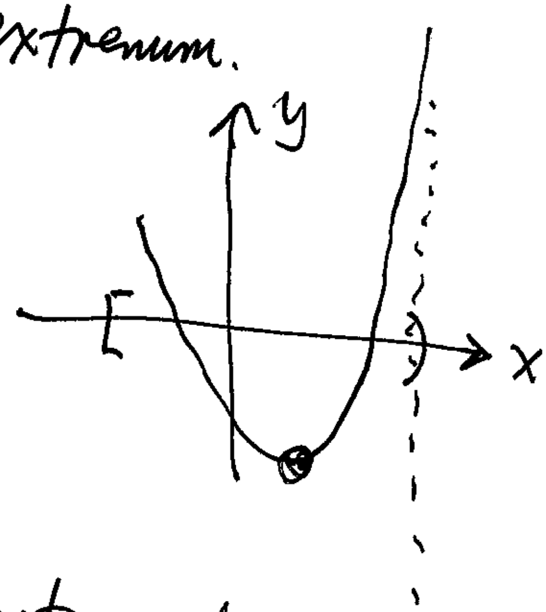
Lesson 25

§4.5 – Max/Min Problems

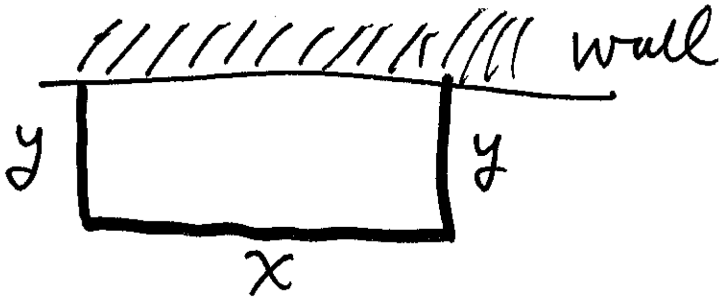
Fact: If there is exactly one local extremum for a cont. $f(x)$ on I
$\Rightarrow$ it is absolute extremum.





$\boxed{\text{Ex1}}$ Find largest rectangular area of a pen that can be enclosed by 20' of fencing, where one side bounded by a wall (no fencing needed)

Soln:



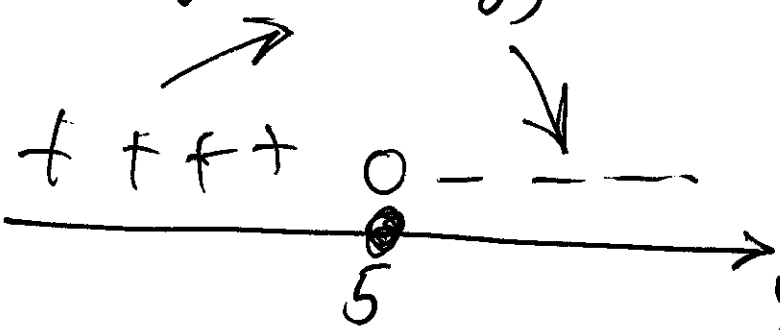
Maximize: $A = xy$ (Objective Function)

Constraint: $x + 2y = 20$

Solve for $x = 20 - 2y$

$\therefore$ Maximize $A(y) = (20-2y)\,y$ ✓

$A'(y) = 4(5-y)$



$A(y)$

$A'(y) = 4(5-y)$

Abs max of A occurs at $y=5$

$A(5) = 50\ ft^2$

Hence $x = 20 - 2(5)$

$x = 10$

Optimization (Max/Min) "Guidelines"

$\boxed{1}$ Read problem

$\boxed{2}$ Draw a picture; label/identify variables

$\boxed{3}$ Identify Objective Fcn (fcn to be extremized)

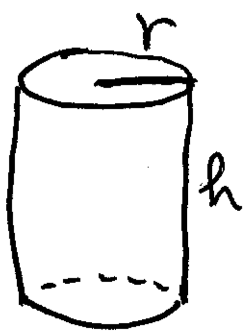
$\boxed{4}$ Identify Constraint(s)

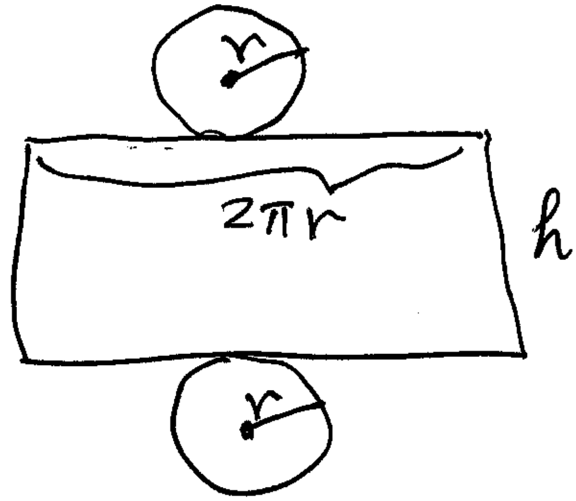
$\boxed{5}$ Use constraints to write obj. fcn as fcn 1 variable.

$\boxed{6}$ Use Calculus to find desired abs. extrema (mostly use 1^{st} Derivative Test)

$\boxed{\text{Ex2}}$ $\boxed{\text{MM-01}}$ An aluminium can is to contain 54 in^3 of cola. What are the dimensions of such a can using least amt of aluminum?

Soln:





Minimize: $A = 2(\pi r^2) + 2\pi r h$

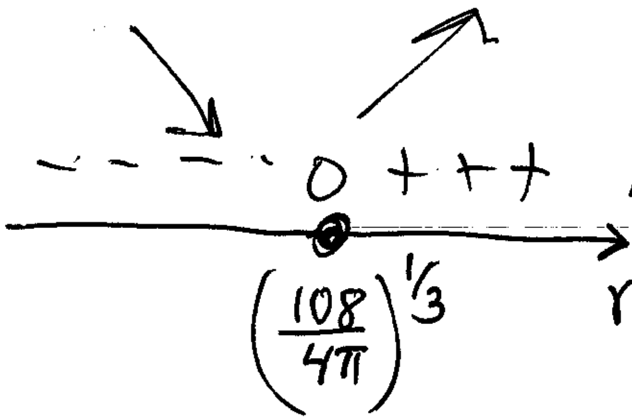
Constraint: $V = \pi r^2 h = 54$

$h = \dfrac{54}{\pi r^2}$

$\therefore$ Minimize $A(r) = 2\pi r^2 + \dfrac{108}{r}$ ✓

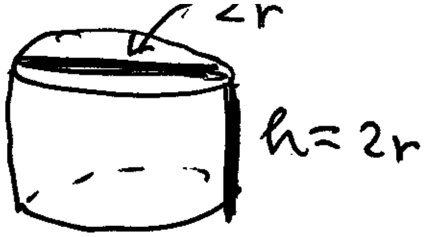
$A(r)$

$A'(r) = \dfrac{4\pi r^3 - 108}{r^2}$



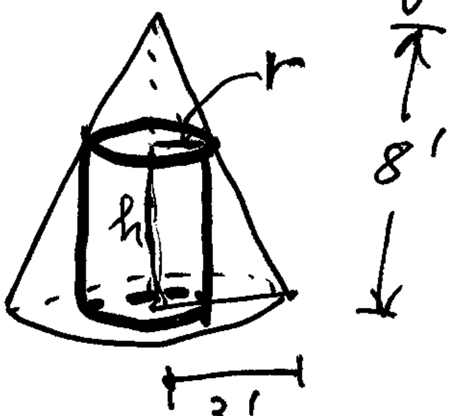
$\therefore$ abs min at $r = \left(\dfrac{108}{4\pi}\right)^{1/3} = \dfrac{3}{\pi^{1/3}}$ ✓

height $h = \dfrac{54}{\pi\left\{\left(\frac{3}{\pi^{1/3}}\right)\right\}^2} = \dfrac{6}{\pi^{1/3}}$ ✓✓ $= 2r$

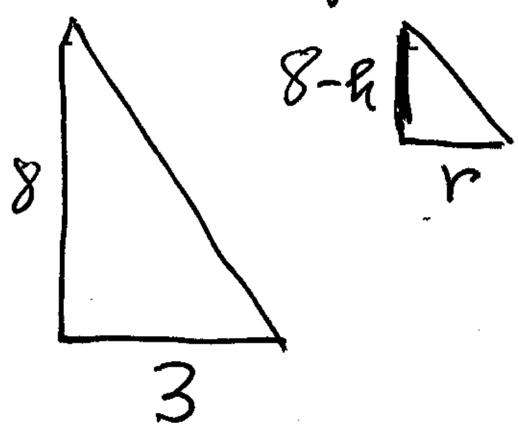


$\boxed{\text{Ex3}}$ $\boxed{\text{MM-05}}$ Find volume of largest cylinder that can be inscribed in a cone of radius 3', height 8'.

Soln:



Maximize: $V = \pi r^2 h$



Similar Δ's

$$\frac{8}{3} = \frac{8-h}{r}$$

$$\Rightarrow h = \frac{24-8r}{3}$$

$$\therefore V = \pi r^2 \left(\frac{24-8r}{3}\right)$$