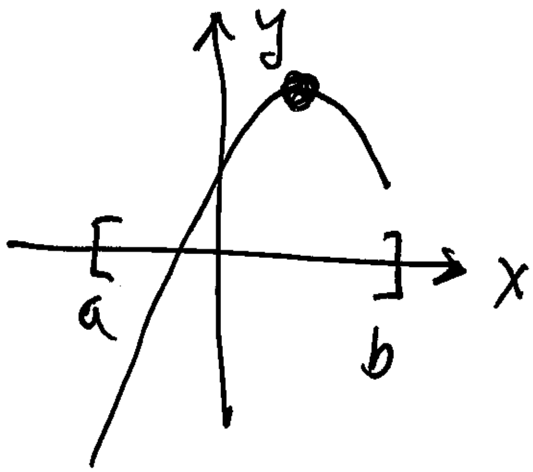
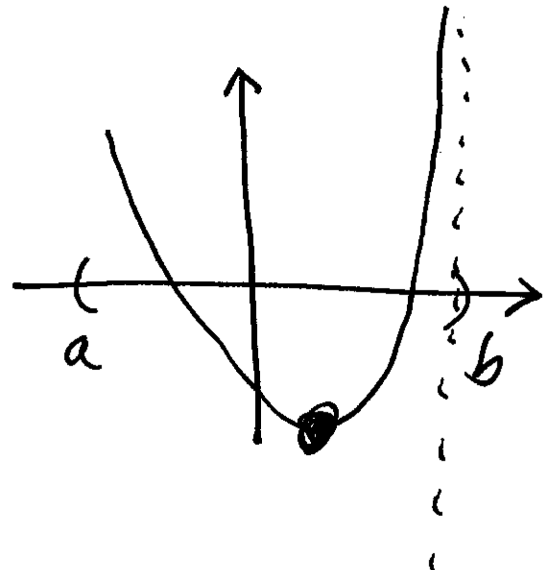
§4.5 – Max/Min Problems

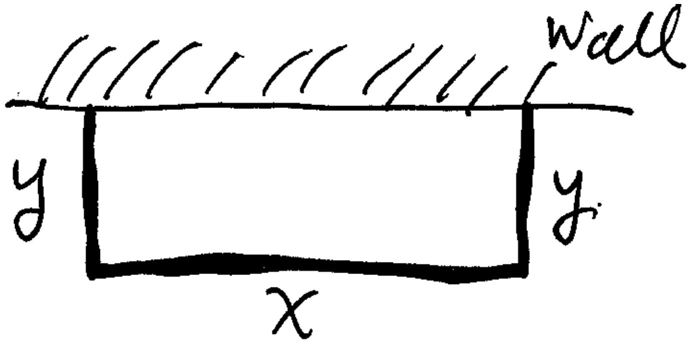
Fact: If there is exactly one local extremum of a cont. $f(x)$ on $I \Rightarrow$ it is an abs. extremum.





$\boxed{\text{Ex1}}$ Find largest rectangular area of a pen that can be enclosed by 20′ of fencing, where one side is bounded by a wall (no fencing needed).

Soln:

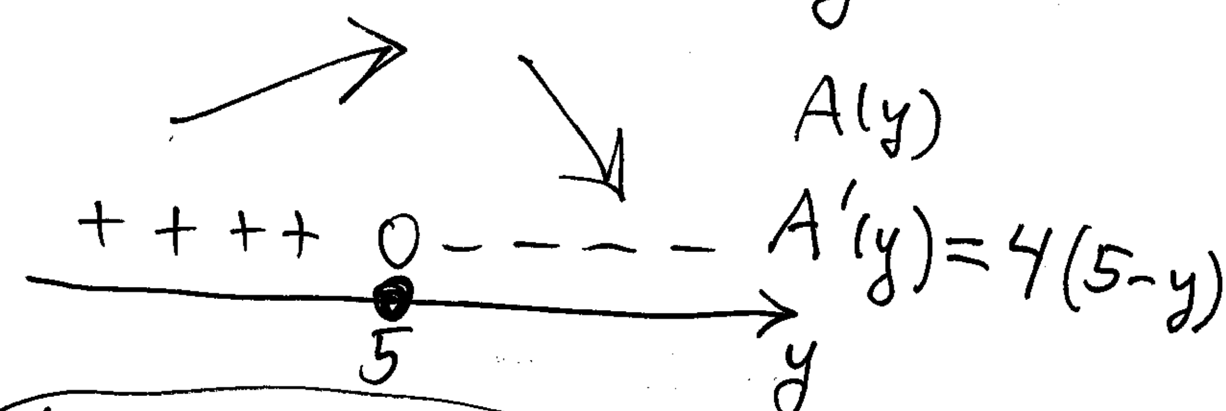


Maximize : $A = xy$ (Objective Function)

Constraint : $x + 2y = 20$

$\Rightarrow \quad x = 20 - 2y$

$\therefore$ Maximize : $A(y) = (20-2y)y$ ✓



$\Rightarrow$ abs. max of A occurs at $y = 5$

$\therefore A(5) = 50 \text{ ft}^2$

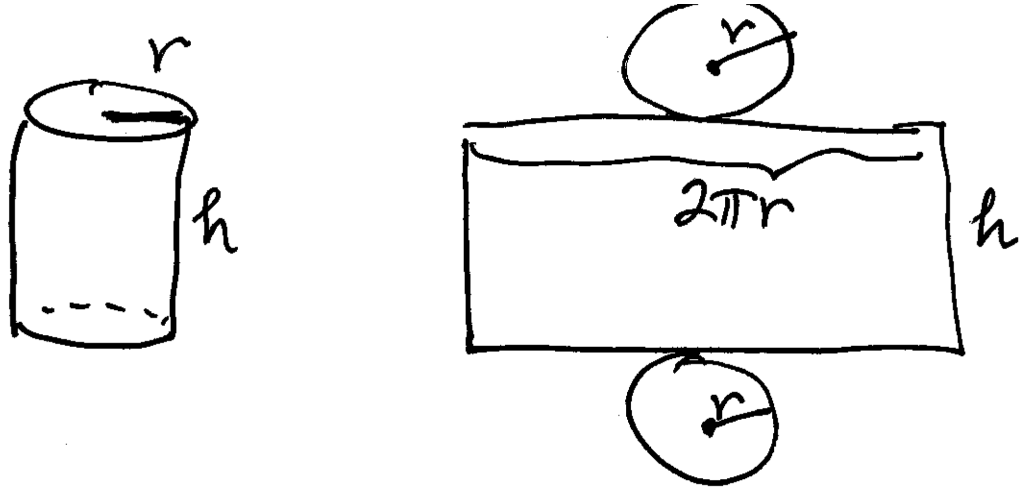
$x = 20 - 2(5) = 10$

Optimization (Max/Min) "Guidelines

1. Read problem.
2. Draw a picture; identify/label variables
3. Identify Objective Function (i.e. fcn to extremized)
4. Identify Constraint(s)
5. Use constraints to write obj. fcn as a fcn of 1 variable
6. Use Calculus to find desired abs. extremum (mostly use 1^{st} Derivative Test)

Ex2 | MM-01 | An aluminum can is to contain 54 in^3 of cola. What are the dimensions of such a can using the least amt of aluminum?

Soln:



Minimize: $A = 2(\pi r^2) + 2\pi r h$

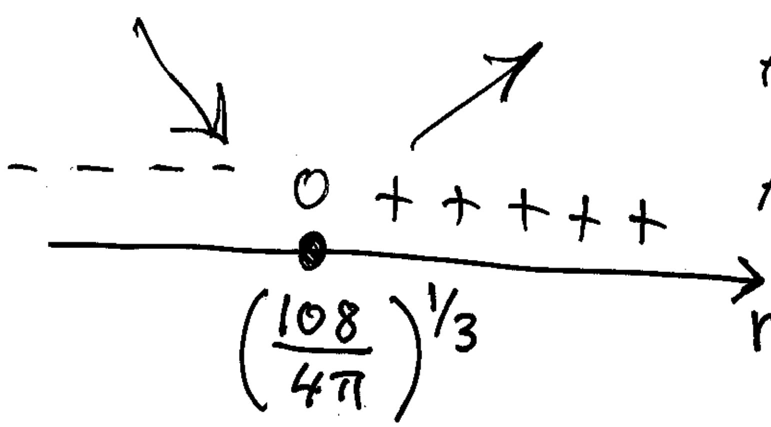
Constraint: $V = \pi r^2 h = 54$

$$h = \frac{54}{\pi r^2}$$

Minimize: $A(r) = 2\pi r^2 + 2\pi r\left(\frac{54}{\pi r^2}\right)$

$A(r)$

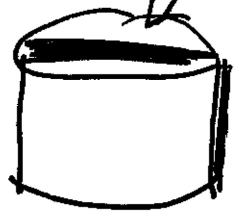
$$A'(r) = \frac{4\pi r^3 - 108}{r^2}$$



∴ abs. min occurs when $r = \left(\frac{108}{4\pi}\right)^{1/3} = \frac{3}{\pi^{1/3}}$

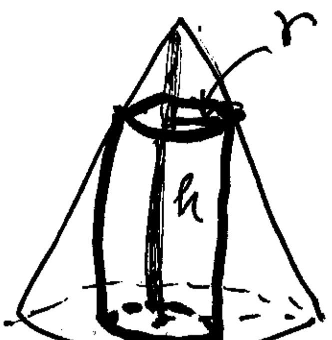
$$h = \frac{54}{\pi\left(\frac{3}{\pi^{1/3}}\right)^2} = \frac{6}{\pi^{1/3}} = 2r$$

$\therefore$

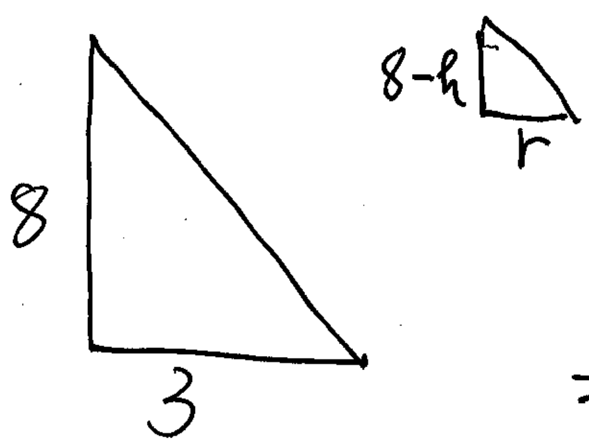


$\boxed{\text{Ex 3}}$ $\boxed{\text{MM-05}}$ Find volume of largest cylinder that can be inscribed in a cone, radius 3', height 8'.

Soln:



Maximize: $V = \pi r^2 h$



Similar Δ's

$$\frac{8}{3} = \frac{8-h}{r}$$

$$\Rightarrow h = \frac{24-8r}{3}$$