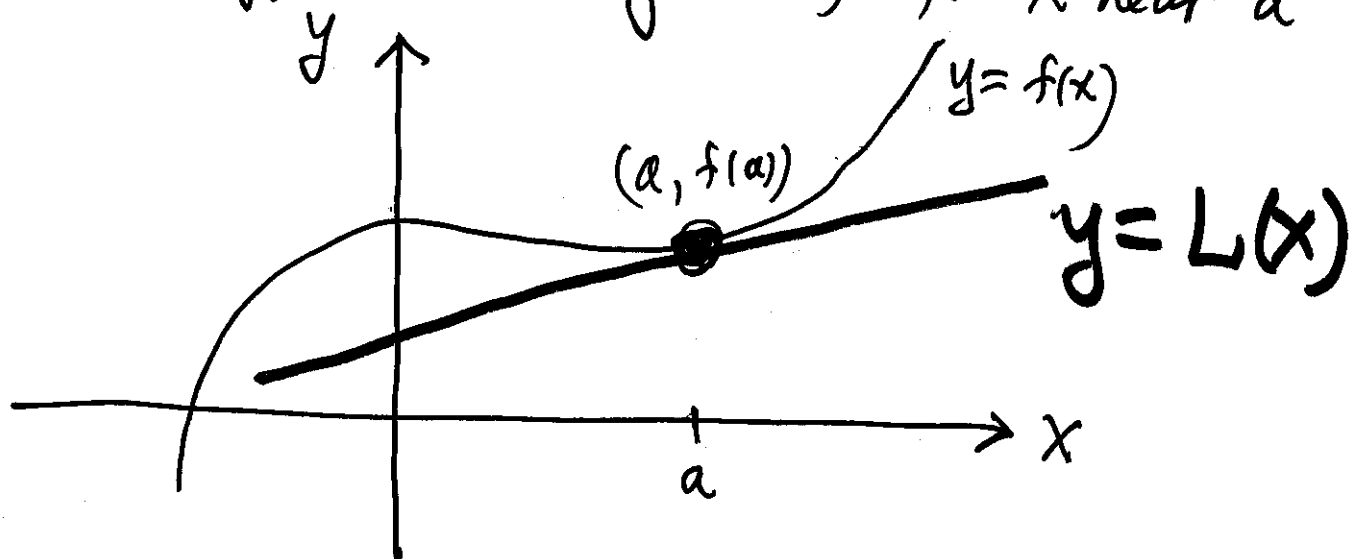


# Lesson 27

①

## §4.6 Linear Approx. + Differentials

Want to approximate  $y = f(x)$  for  $x$  near  $a$



Linear Approximation to  $f(x)$  at  $a$ :

$$f(x) \approx L(x) = f(a) + f'(a)(x-a)$$

for  $x$  near  $a$

Remark: If  $f''(a) > 0 \Rightarrow$  approximations are underestimated  
 If  $f''(a) < 0 \Rightarrow$  approx. are overestimated

**Ex 1** Approximate:

(2)

(a)  $\frac{1}{\sqrt{4.5}}$  ;  $f(x) = \frac{1}{\sqrt{x}}$  let  $a=4$

$$(f'(x) = -\frac{1}{2x^{3/2}})$$

$$\therefore f(x) \approx f(4) + f'(4)(x-4)$$

$$\boxed{\frac{1}{\sqrt{x}} \approx \frac{1}{2} - \frac{1}{16}(x-4)} \quad \text{for } x \text{ near } 4$$

$$\Rightarrow \frac{1}{\sqrt{4.5}} \approx \frac{1}{2} - \frac{1}{16}(4.5-4) = \frac{30}{64} = 0.46875$$

$\nwarrow$   $0.471404\dots$

$\swarrow$  close!

Since  $f''(4) > 0$



$\Rightarrow$  Linear approx is underestimate.

Also  $\frac{1}{\sqrt{3.4}} \approx \frac{1}{2} - \frac{1}{16}(3.4-4) = 0.5375$

"  
 $0.5423\dots$

⑥  $\sin 5^\circ$  ;  $f(x) = \sin x$  let  $a=0$  ⑦  
 $(f'(x) = \cos x)$   
 $x$  must be in radians

$5^\circ$  is  $(5) \left(\frac{\pi}{180}\right) = \frac{\pi}{36}$  radians

$\therefore f(x) \approx f(a) + f'(a)(x-a)$  a=0

$\sin x \approx 0 + 1(x-0)$

$\Rightarrow \boxed{\sin x \approx x}$  for  $x$  near 0

$\therefore \sin \frac{\pi}{36} \approx \frac{\pi}{36} = 0.08727$

0.087155743

⑦  $f(1.2)$  if

$x$	$f(x)$	$f'(x)$
1	4	-3
2	0	1

$\therefore f(1.2) \approx f(1) + f'(1)(1.2-1)$   
 $= 4 + (-3)(0.2) \checkmark$

Recall  $\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a} = f'(a)$

(4)

$\Rightarrow \frac{f(x) - f(a)}{x - a} \approx f'(a) \quad x \text{ near } a$

$f(x) - f(a) \approx f'(a)(x - a)$

$f(x) \approx f(a) + f'(a)(x - a) \checkmark$

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$y = f(x)$

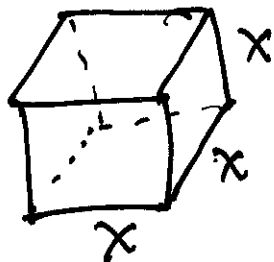
let  $\Delta x = x - a$  (change in  $x$ )

$\Delta y = f(x) - f(a)$  (change in  $y$ )

$\Delta y \approx f'(a) \Delta x$

Approximate  
Change Formula

EX2



$$V = x^3$$

5

Estimate change in volume when  $x$  changes from 2 to 2.1

$a$   $\uparrow$   $a + \Delta x$

$$\Delta V \approx V'(a) \Delta x$$
$$= 3a^2 \Delta x$$

$\Delta x = 0.1$

$$\therefore \Delta V \approx 3(2)^2(0.1) = 1.2 \text{ m}^3$$

//

$$V(2.1) - V(2)$$

$$= (2.1)^3 - (2)^3 = 1.261 \text{ m}^3$$

⑥

Def:  $y = f(x)$

the differential of  $x$ ,  $dx$ , is a small change in  $x$ .

the differential of  $y$  is  $\boxed{dy = f'(x) dx}$

Next time :-

$$\Delta x = dx$$

$$\Delta y \approx dy \quad \checkmark$$