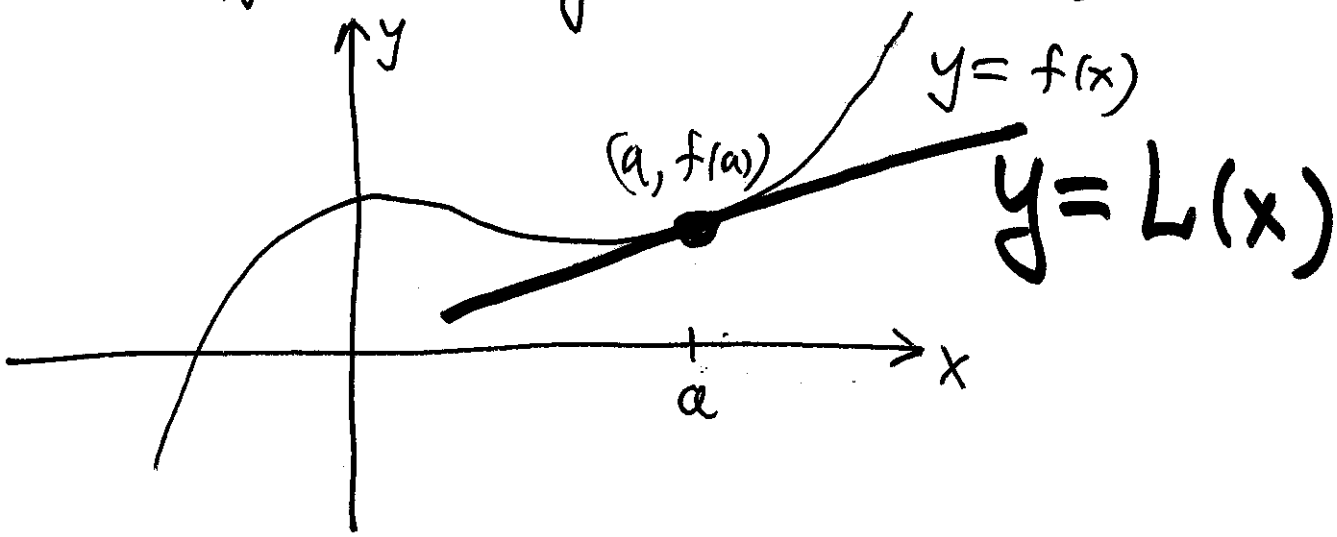
Lesson 27

§4.6 Linear Approx. & Differentials

Want to approximate $y = f(x)$ near a



Linear Approximation to $f(x)$ at a:

$$f(x) \approx L(x) = f(a) + f'(a)(x-a)$$

for x near a

Remark: If $f''(a) > 0 \Rightarrow$ approximation is an underestimation

If $f''(a) < 0 \Rightarrow$ approx. is an overestimation

Ex 1 Approximate

(a) $\frac{1}{\sqrt{4.5}}$; $f(x)=\frac{1}{\sqrt{x}}$ let $a=4$

$\left(f'(x)=-\frac{1}{2x^{3/2}}\right)$

$\therefore\ f(x)\approx f(a)+f'(a)(x-a)$

$\frac{1}{\sqrt{x}}\approx f(4)+f'(4)(x-4)$

$\boxed{\frac{1}{\sqrt{x}}\approx\frac{1}{2}-\frac{1}{16}(x-4)}$ for x near 4

$\Rightarrow \frac{1}{\sqrt{4.5}}\approx\frac{1}{2}-\frac{1}{16}(4.5-4)=\frac{30}{64}=0.46875$

$\downarrow$

0.471404...

Also can estimate $\frac{1}{\sqrt{3.4}}\approx\frac{1}{2}-\frac{1}{16}(3.4-4)$

$=0.5375$

$\swarrow$

0.5423....

(b) $\sin 5^\circ$; $f(x) = \sin x$ let $a = 0$

$(f'(x) = \cos x)$

x must be in radians!

5° is $5\left(\frac{\pi}{180}\right) = \frac{\pi}{36}$ radians

$\therefore f(x) \approx f(a) + f'(a)(x-a)$ $\quad (a = 0)$

$\sin x \approx 0 + (1)(x-0)$

$$\boxed{\sin x \approx x}$$ for x near 0

Hence $\sin \frac{\pi}{36} \approx \frac{\pi}{36} = 0.08727$

$\uparrow$

0.087155743

(c) $f(1.2)$ if

x	$f(x)$	$f'(x)$
1	4	-3
2	0	1

$f(1.2) \approx f(1) + f'(1)(1.2-1)$

$= 4 + (-3)(0.2)$ ✓

Recall $\lim_{x \to a} \frac{f(x) - f(a)}{x - a} = f'(a)$

$\Rightarrow \quad \frac{f(x) - f(a)}{x - a} \approx f'(a)$ $\quad x$ near a

$f(x) - f(a) \approx f'(a)(x - a)$

$f(x) \approx f(a) + f'(a)(x - a)$ ✓

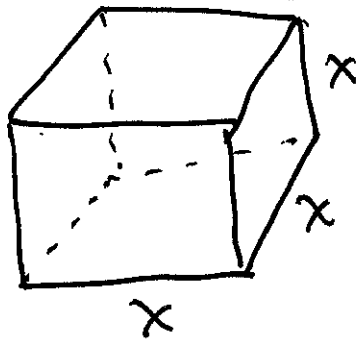
Let $y = f(x)$

let $\Delta x = x - a$ (change in x)

$\Delta y = f(x) - f(a)$ (change in y)

$\boxed{\Delta y \approx f'(a)\,\Delta x}$ Approximate Change Formula

Ex 2



$V(x) = x^3$

$V' = 3x^2$

Estimate change in volume when x changes from 2 to 2.1

(a = 2, $a + \Delta x$ = 2.1)

$\Delta x = 0.1$

$$\Delta V \approx V'(a)\,\Delta x$$

$$\Delta V \approx 3(2)^2(0.1) = 1.2 \text{ m}^3$$

$$V(2.1) - V(2) = (2.1)^3 - (2)^3 = \underline{1.261 \text{ m}^3}$$

exact change

Def: $y = f(x)$

the differential of x, dx, is a small change in x

the differential of y is $\boxed{dy = f'(x)\,dx}$

Next time: $\Delta x, \Delta y, dx, dy$