Lesson 29 ← last lesson on Exam #3

Last time: Which grows faster?

x^2, $e^{0.001x}$ $\Rightarrow \lim\limits_{x\to\infty} \dfrac{x^2}{e^{0.001x}} = 0$

$e^{0.001x}$ grows faster

LR 2 times

$e^x + 2^x$, $5e^x$ ← comparable growth!

$$\lim_{x\to\infty} \frac{e^x + 2^x}{5e^x} = \lim_{x\to\infty}\left(\frac{e^x}{5e^x} + \frac{2^x}{5e^x}\right)$$

$$= \lim_{x\to\infty}\left(\frac{1}{5} + \frac{1}{5}\left(\frac{2}{e}\right)^x\right) = \frac{1}{5}$$

$\boxed{\S 4.9}$ Antiderivatives

__Def:__ If $F'(x) = f(x)$ for all in x in $\underline{I}$

then $F(x)$ is __antiderivative__ of $f(x)$

For eg $f(x) = 3x^2 + 4\cos x$

an antiderivative of $f(x)$ is $F(x) = x^3 + 4\sin x$

If $g(x)$ is any other antiderivative of $f(x)$ (i.e. $g'(x) = f(x)$) then

$$\{g(x) - F(x)\}' = g'(x) - F'(x) = f(x) - f(x) = 0$$

$\Rightarrow g(x) - F(x) = C$. Hence

<u>Thm</u> If $F(x)$ is an antiderivative of $f(x)$ on I $\Rightarrow$ all antiderivatives of $f(x)$ have the form

$$F(x) + C.$$

For eg: if $f(x) = 3x^2 + 4\cos x$

<u>all</u> antiderivatives $g(x) = x^3 + 4\sin x + C$

Notation: $\boxed{\int f(x)\,dx}$ = collection of all antiderivatives of $f(x)$

indefinite integral of $f(x)$

integrand

x is dummy variable

For eg $\int (7x^3 - 1)\,dx = \frac{7}{4}x^4 - x + C$

Facts:

① $\int k f(x)\,dx = k \int f(x)\,dx$

② $\int \{f(x) + g(x)\}\,dx = \int f(x)\,dx + \int g(x)\,dx$

Note $\frac{d}{dx}\left\{ \frac{x^{p+1}}{p+1} \right\} = x^p \qquad (p \neq -1)$

Hence,

Power Rule For Integrals

$$\boxed{\int x^p\,dx = \frac{x^{p+1}}{p+1} + C} \quad p \neq -1$$

For eg, $\displaystyle\int \frac{8}{\sqrt{x}}\,dx = 8\int x^{-\frac{1}{2}}\,dx$

$$= 8\left[\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C_1\right]$$

$$= 8\left[2\sqrt{x} + C_1\right]$$

$$= \underline{16\sqrt{x} + C}$$ diff. to check.

$\boxed{\text{Ex 1}}$ Evaluate

ⓐ $\int \left(6x + 2\sin x + \frac{4}{x}\right) dx$

$= 6\left(\frac{x^2}{2}\right) + 2\left(-\cos x\right) + 4\left(\ln|x|\right) + C$

ⓑ $\int (2r-3)^2 \, dr = \int \left(4r^2 - 12r + 9\right) dr$

$= \frac{4r^3}{3} - \frac{12r^2}{2} + 9r + C$

$\boxed{\text{Ex2}}$ If slope of $y = f(x)$ at (x, y) is $-3x^2 + 1$ and f passes through $(2, -1)$, find $f(x)$.

Soln: $f'(x) = -3x^2 + 1$ IVP

$f(2) = -1$ ← initial condition. Initial Value Problem

Antiderivative $f(x) = -x^3 + x + C$

$\therefore -1 = f(2) = -8 + 2 + C = -6 + C \Rightarrow C = 5$

$f(x) = -x^3 + x + 5$