Lesson 29 ← last lesson on Exam #3

Last time: Which grows faster?

x^2, $e^{0.001x}$: $\lim_{x\to\infty} \dfrac{x^2}{e^{0.001x}} \overset{\text{LR 2 times}}{=} 0$

⇒ $e^{0.001x}$ grows faster

$e^x + 2^x$, $5e^x$ ← comparable!

$$\lim_{x\to\infty} \frac{e^x+2^x}{5e^x} = \lim_{x\to\infty}\left(\frac{e^x}{5e^x} + \frac{2^x}{5e^x}\right)$$

$$= \lim_{x\to\infty}\left(\frac{1}{5} + \frac{1}{5}\left(\frac{2}{e}\right)^x\right) = \frac{1}{5}$$

§4.9 Antiderivatives

<u>Def</u>: If $F'(x) = f(x)$ for all x in I
then $F(x)$ is an <u>antiderivative</u> of $f(x)$

For eg, if $f(x) = 3x^2 + 4\cos x$
then $F(x) = x^3 + 4\sin x$ is an antiderivative of $f(x)$
since $F'(x) = f(x)$

If $g(x)$ is any other antiderivative of f (i.e $g'(x) = f(x)$)

$$\Rightarrow \{g(x) - F(x)\}' = g'(x) - F'(x) = f(x) - f(x) = 0$$

$$\Rightarrow g(x) - F(x) = C \quad \therefore g(x) = F(x) + C$$

<u>Thm</u>: If $F(x)$ is any antiderivative of $f(x)$ on I
$\Rightarrow$ <u>all</u> antiderivatives of $f(x)$ have form

$$F(x) + C$$

For eg, if $f(x) = 3x^2 + 4\cos x \Rightarrow x^3 + 4\sin x + C$
all antiderivatives

Notation: $\boxed{\int f(x)\,dx}$ = collection of all antiderivatives of $f(x)$

indefinite integral of $f(x)$

integrand

x is a dummy variable

For eg, $\int (7x^3 - 1)\,dx = \frac{7x^4}{4} - x + C$

Facts: ① $\int k\,f(x)\,dx = k \int f(x)\,dx$

② $\int \{f(x) + g(x)\}\,dx = \int f(x)\,dx + \int g(x)\,dx$

Note: $\frac{d}{dx}\left(\frac{x^{p+1}}{p+1}\right) = x^p \qquad (p \neq -1)$

Hence,

Power Rule for Integrals

$$\boxed{\int x^p\,dx = \frac{x^{p+1}}{p+1} + C} \quad p \neq -1$$

For eg, $\displaystyle\int \frac{8}{\sqrt{x}}\,dx = 8\int x^{-\frac{1}{2}}\,dx$

$$= 8\left[\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C_1\right]$$

$$= 8\left[2\sqrt{x} + C_1\right]$$

$$= \underline{\underline{16\sqrt{x} + C}}$$

diff.

Ex 1 Evaluate

(a) $\int \left(6x + 2\sin x + \frac{4}{x}\right) dx$

$= 6\left(\frac{x^2}{2}\right) + 2(-\cos x) + 4\left(\ln|x|\right) + C$

(b) $\int (2r-3)^2 dr = \int \left(4r^2 - 12r + 9\right) dr$

$= \frac{4r^3}{3} - \frac{12r^2}{2} + 9r + C$

Ex 2 If slope of $y = f(x)$ at (x,y) is $-3x^2+1$ and f passes through $(2,-1)$, find $f(x)$.

Soln: $\left.\begin{array}{l} f'(x) = -3x^2+1 \\ f(2) = -1 \end{array}\right\}$ Initial Value Problem

$f(2) = -1$ ← initial condition.

Antiderivative is $f(x) = -x^3 + x + C$

so $-1 = f(2) = -6 + C$

$C = 5$