Last time

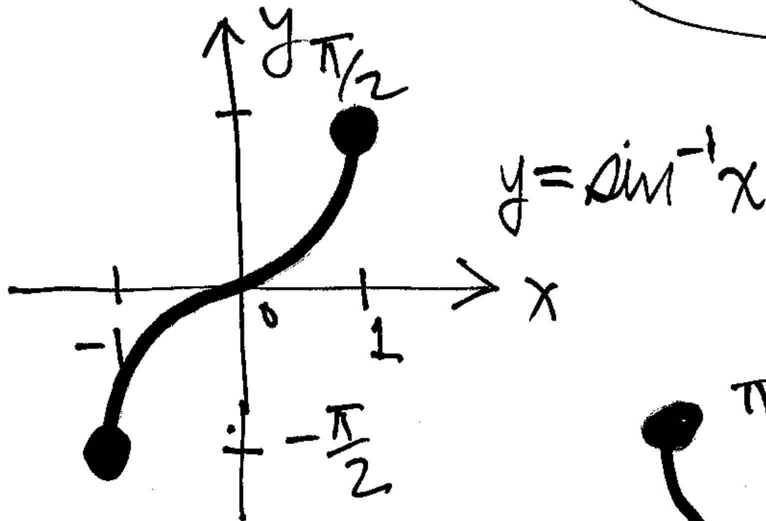
① Solve $\cos x = \frac{1}{2} \Rightarrow x = \frac{\pi}{3} + 2\pi k$

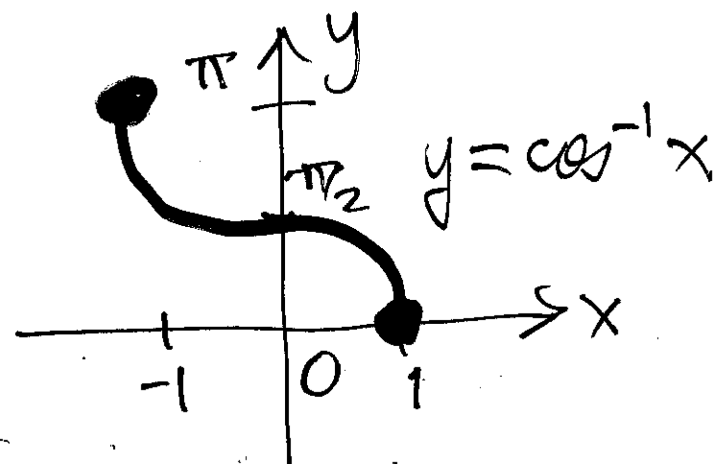
$k = 0, \pm 1, \pm 2, \dots$

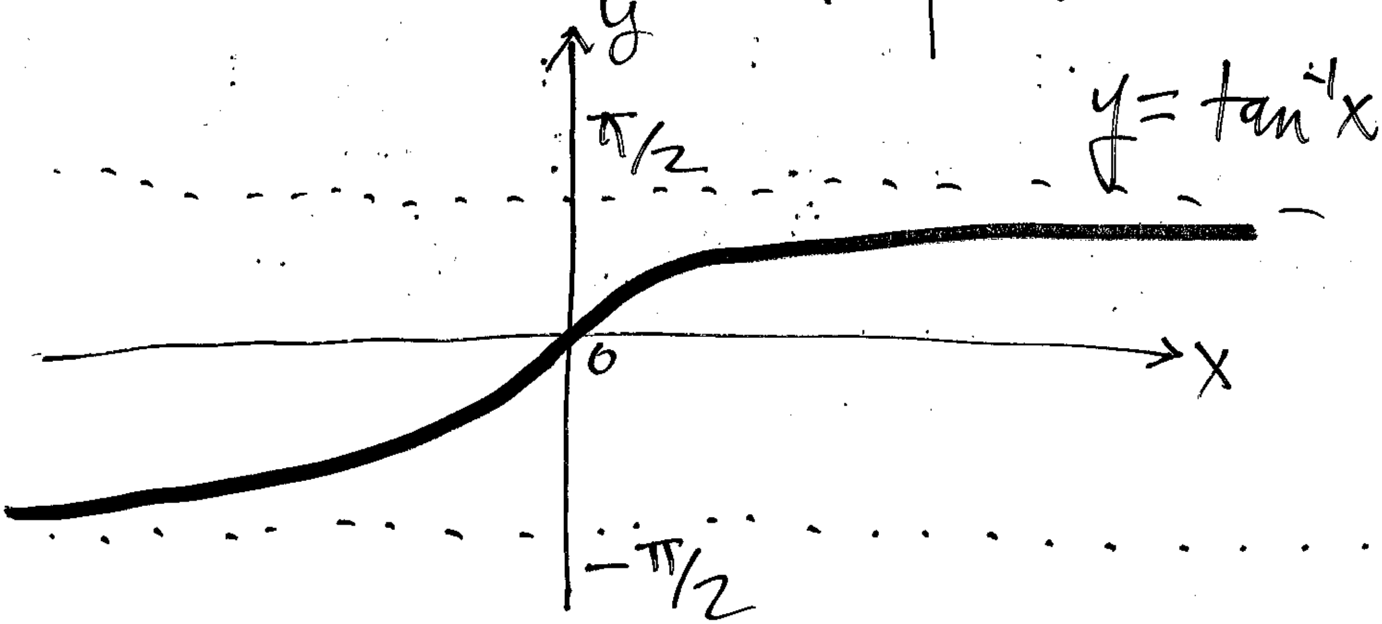
AND $x = -\frac{\pi}{3} + 2\pi m$

$m = 0, \pm 1, \pm 2, \dots$

②







③ $\boxed{Ex}$ $\cos\left(\cos^{-1}\frac{\sqrt{3}}{2}\right) = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

$\sin^{-1}\left(\sin\frac{3\pi}{2}\right) = \sin^{-1}(-1) = -\frac{\pi}{2}$

$\cos^{-1}(2)$ Impossible

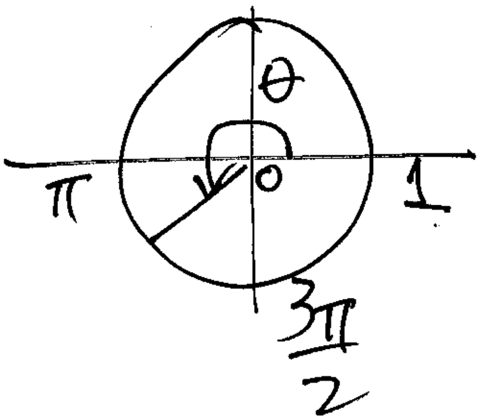
④ If $\sin\theta = -\frac{5}{13}$ and $\pi < \theta < \frac{3\pi}{2}$

find $\cos\theta$.

Soln: $\sin^2\theta + \cos^2\theta = 1$

$\left(-\frac{5}{13}\right)^2 + \cos^2\theta = 1$

$\cos^2\theta = \frac{144}{169} \Rightarrow \cos\theta = \pm\frac{12}{13}$

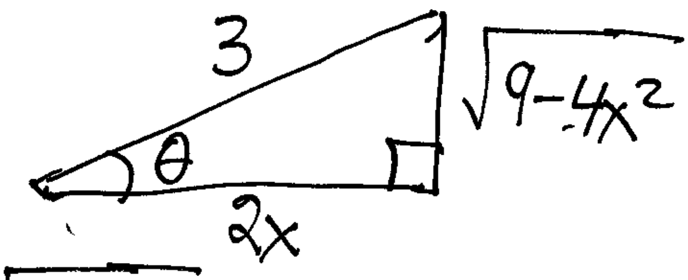


$\therefore \cos\theta = -\frac{12}{13}$ ✓

5 Simplify $\tan\left(\underbrace{\cos^{-1}\left(\frac{2x}{3}\right)}_{\theta}\right)$

$\theta = \cos^{-1}\left(\frac{2x}{3}\right)$

$\cos\theta = \frac{2x}{3}$



$\therefore \tan\theta = \frac{\sqrt{9-4x^2}}{2x}$ ✓

§§ 2.1 + 2.2 — Limits

If $s(t)$ = displacement of object

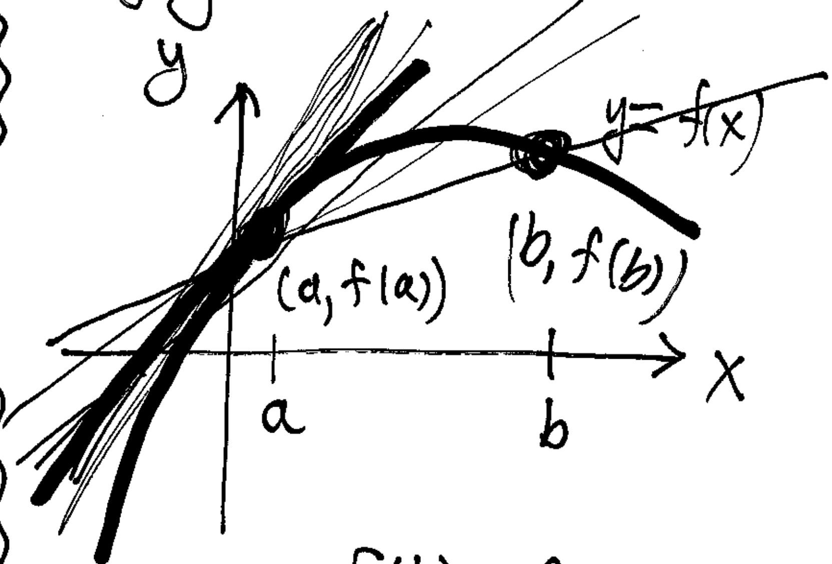
$$v_{av} = \frac{s(b) - s(a)}{b-a}$$

average velocity over $[a, b]$

$$v_{ins} = \lim_{b \to a} v_{av}$$

instantaneous velocity

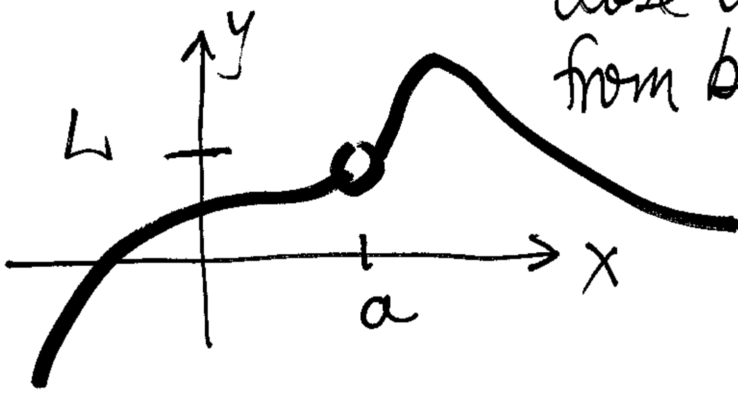
If $y = f(x)$ is function



$$m_{sec} = \frac{f(b) - f(a)}{b-a}$$

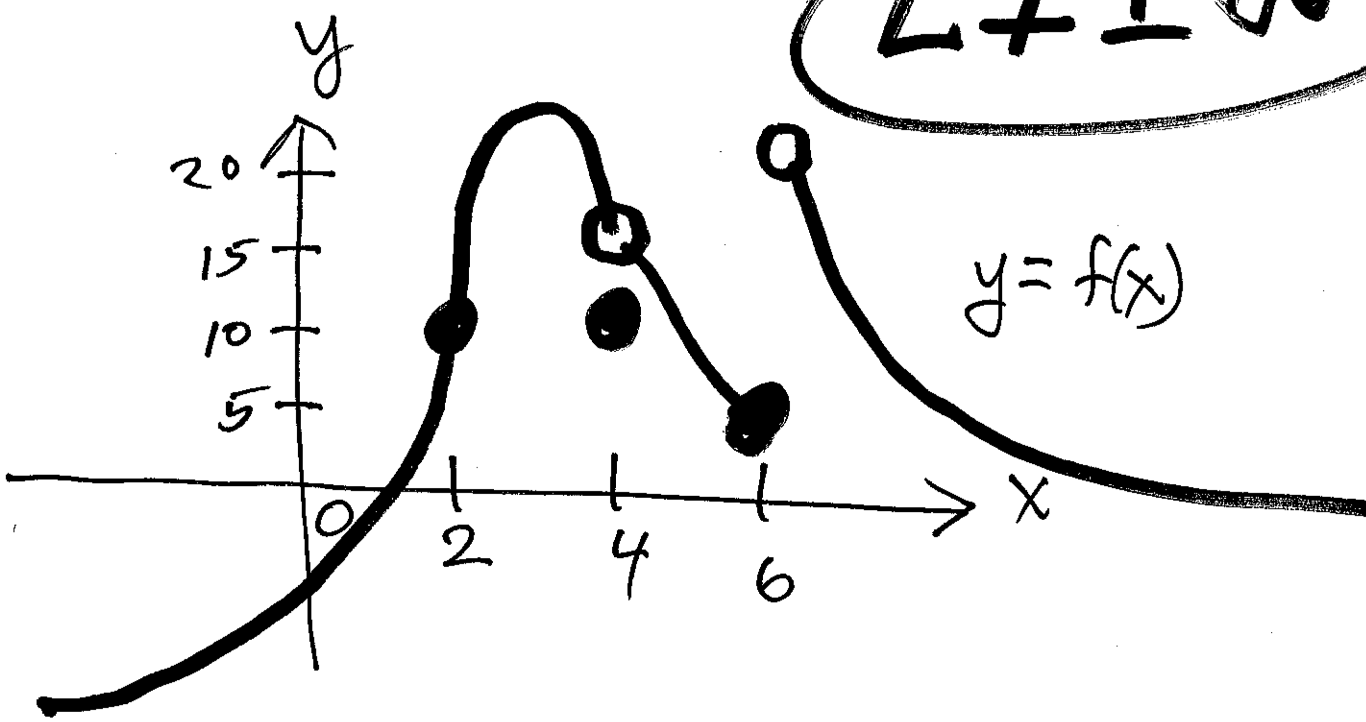
$$m_{tan} = \lim_{b \to a} m_{sec}$$

$\boxed{\lim_{x \to a} f(x) = L}$ means $f(x)$ is arbitrarily close to L for all x suff. close to a (but not equal to a) from both sides of a



$y = f(x)$

$L \neq \pm\infty$



$\lim_{x \to 2} f(x) = 10$

$\lim_{x \to 4} f(x) = 15$

$\lim_{x \to 6} f(x) =$ Does Not Exist (DNE)

Right-sided limits: $\boxed{\lim_{x \to a^+} f(x) = L}$

means $f(x)$ is arbitrarily close to L for all $x > a$ suff. close to a

Left-sided limits: $\boxed{\lim_{x \to a^-} f(x) = L}$

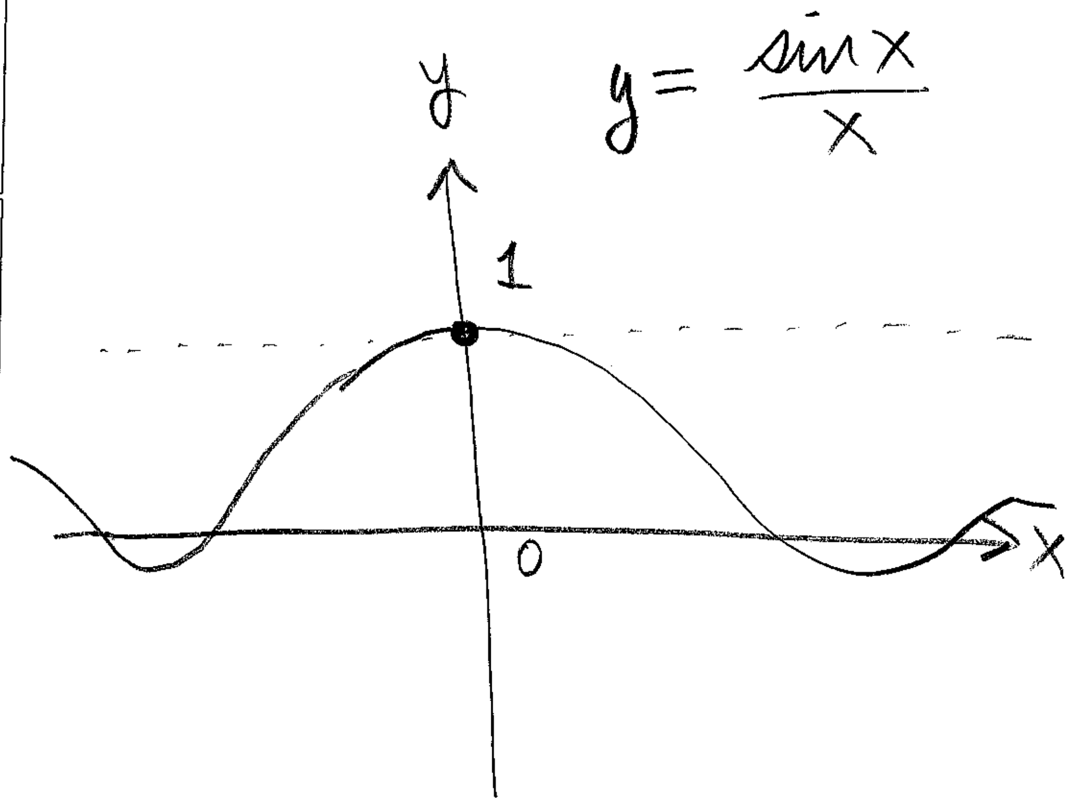
means $f(x)$ is arb. close to L for all $x < a$ suff. close to a

Theorem: $\lim_{x \to a} f(x) = L$ exists

$$\iff \lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x) = L$$

$$\lim_{x \to 0} \frac{\sin x}{x} = \boxed{1}$$

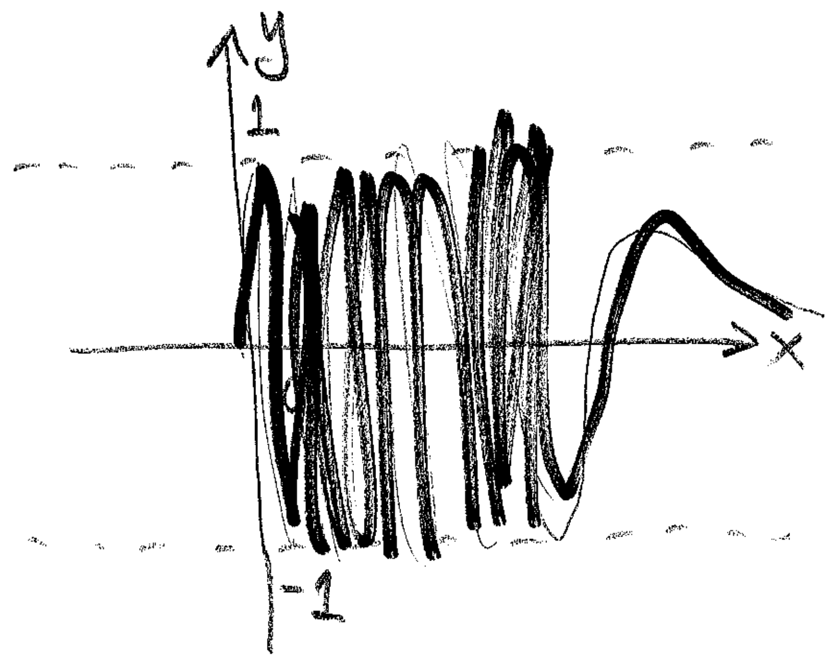
x (*radians*)	$\frac{\sin x}{x}$
1.0	0.84147
0.1	0.99833
0.01	0.99998
0.001	0.99999
0.0001	0.99999
0.00001	0.99999
−1.0	0.84147
−0.1	0.99833
−0.01	0.99998
−0.001	0.99999
−0.0001	0.99999
−0.00001	0.99999



$$\lim_{x\to 0^+} \sin\left(\frac{\pi}{x}\right) = \boxed{\text{DNE}}$$

x (*radians*)	$\sin\left(\frac{\pi}{x}\right)$
1	1.225×10^{-16}
1/2	-2.4503×10^{-16}
1/8	-9.80119×10^{-16}
1/32	-392048×10^{-15}
1/64	-7.84095×10^{-15}
2/5	1
2/9	1
2/13	1
2/3	−1
2/7	−1
2/11	−1

$y = \sin\left(\frac{\pi}{x}\right)$



III

$$\lim_{x\to 0}\left[\frac{1}{x}-\frac{1}{x}\left(\sqrt{1-6x}\right)\right]=\boxed{3}$$