

# FRIDAY LECTURE

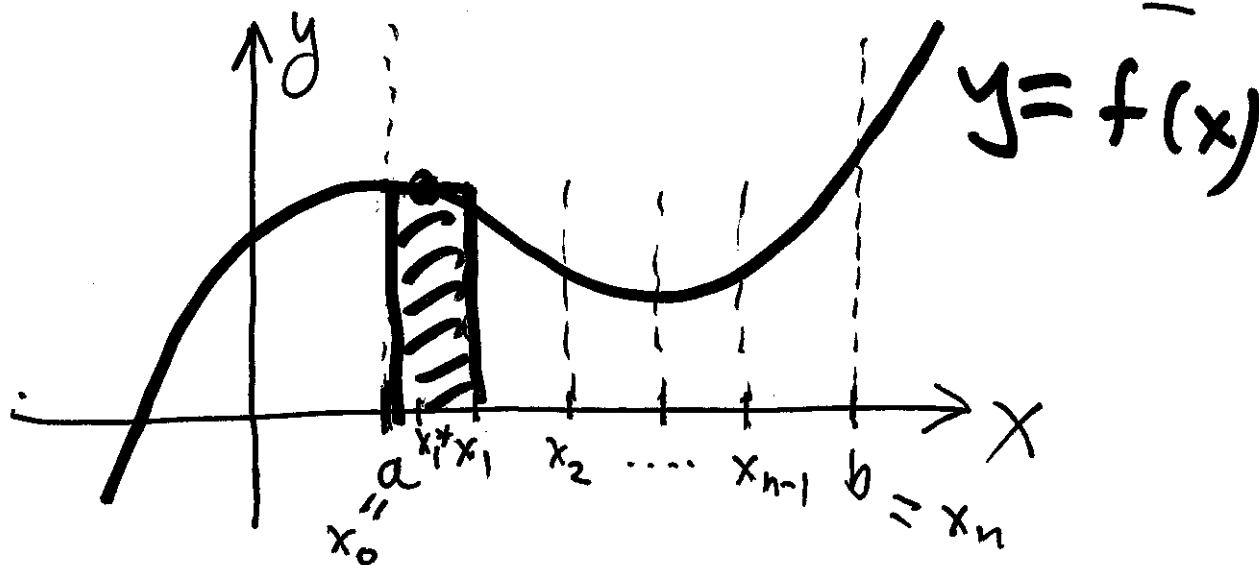
①

## in ELLIOTT HALL OF MUSIC

Lesson 30 ← not on exam

### §5.1 - Approximating Areas Under Curves

$y = f(x)$ ,  $I = [a, b]$  and  $f(x) \geq 0$  on  $I$



Partition  $[a, b]$  into  $n$  equal subintervals

$$\text{let } \Delta x = \frac{b-a}{n} ; \quad x_k = x_0 + k \Delta x$$



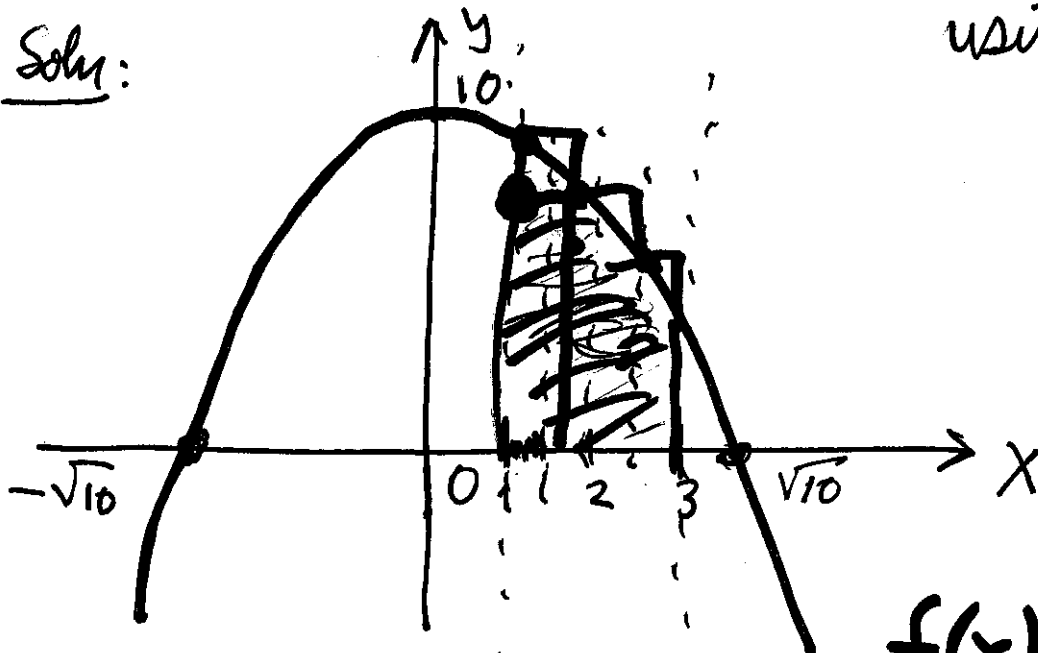
**Ex1** Approx. area under

3

$y = f(x) = 10 - x^2$  between  $x=1$  and  $x=3$

Soln:

using Riemann  
Sums with  
 $n=4$



$$f(x) = 10 - x^2$$

$$\Delta x = \frac{3-1}{4} = \frac{1}{2}$$

$$\left[1, \frac{3}{2}\right], \left[\frac{3}{2}, 2\right], \left[2, \frac{5}{2}\right], \left[\frac{5}{2}, 3\right]$$

$$\begin{aligned} \text{Left Riemann Sum} &= f(1)\Delta x + f\left(\frac{3}{2}\right)\Delta x + f(2)\Delta x + f\left(\frac{5}{2}\right)\Delta x \\ &= 13.25 \checkmark \Rightarrow \text{get over estimation} \end{aligned}$$

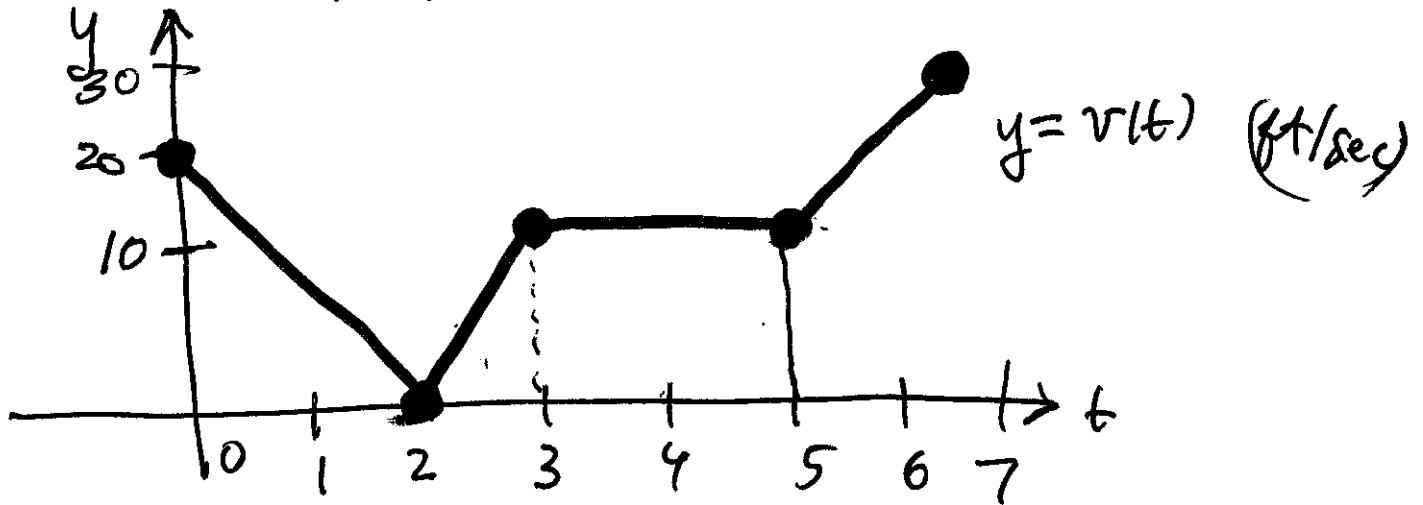
$$\begin{aligned} \text{Right Riemann Sum} &= f\left(\frac{3}{2}\right)\Delta x + f(2)\Delta x + f\left(\frac{5}{2}\right)\Delta x + f(3)\Delta x \\ &= 9.25 \checkmark \text{ underestimate} \end{aligned}$$

$$\begin{aligned} \text{midpt Riemann sum} &= f\left(\frac{1+\frac{3}{2}}{2}\right)\Delta x + \dots + f\left(\frac{\frac{5}{2}+3}{2}\right)\Delta x \\ &= 11.375 \checkmark \end{aligned}$$

$$\begin{aligned} \text{Exact Area} \\ &= \frac{34}{3} = 11.333\dots \end{aligned}$$

(4)

**Ex2** Velocity of object:



Find object's displacement from

$t=0$  to  $t=3$ :

$$D = \frac{1}{2}(2)(20) + \frac{1}{2}(1)(10) = 25$$

$t=2$  to  $t=5$ :

$$D = \frac{1}{2}(3+2)(10) = 25$$