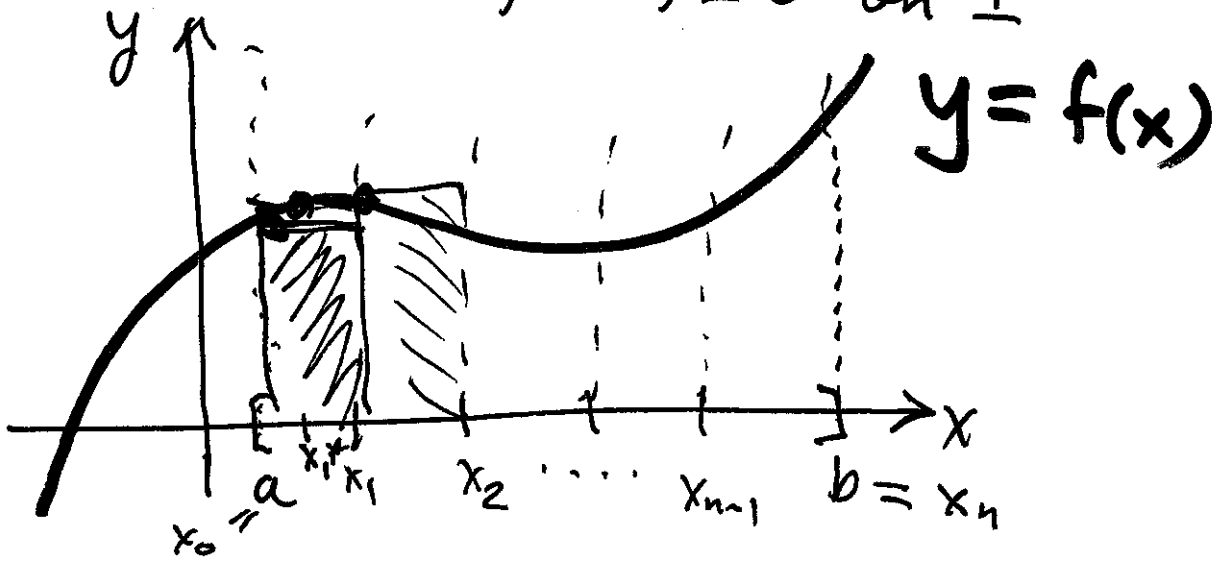
FRIDAY LECTURE in ELLIOTT HALL OF MUSIC

Lesson 30 ← not on Exam #3

§5.1 Approximating Areas Under Curves

$y = f(x)$, $I = [a, b]$, $f(x) \geq 0$ on I



Partition $[a,b]$ into n equal parts: $\Delta x = \frac{b-a}{n}$

$x_k = x_0 + k\Delta x$. Get n subintervals

$[x_0, x_1], [x_1, x_2], \cdots [x_{n-1}, x_n]$

$x_0 = a$, $x_n = b$

let x_k^* be a point in $[x_{k-1}, x_k]$

$$\Rightarrow \underbrace{f(x_1^*)\Delta x + f(x_2^*)\Delta x + \cdots + f(x_n^*)\Delta x}_{\text{Riemann Sum of } f \text{ on } [a,b]}$$

① if $x_k^* = x_{k-1} \Rightarrow$ left Riemann sum of f

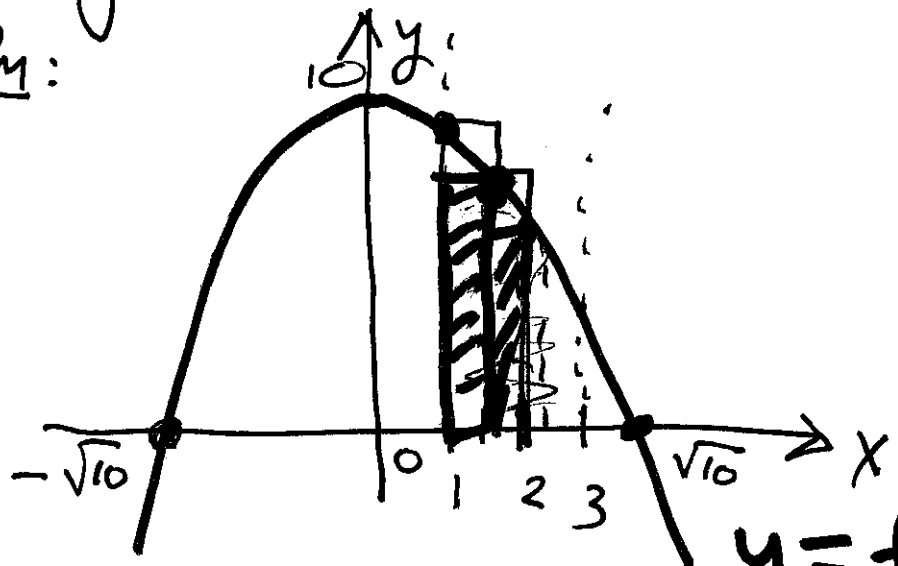
② if $x_k^* = x_k \Rightarrow$ right Riemann sum of f

③ if $x_k^* = \frac{x_{k-1} + x_k}{2} \Rightarrow$ midpt Riemann sum of f

Ex 1 Approx. area under

$y = f(x) = 10 - x^2$ between $x = 1$ and $x = 3$ using Riemann sums with $n = 4$.

Soln:



$y = f(x) = 10 - x^2$

$\Delta x = \frac{3-1}{4} = 1/2 \Rightarrow [1, \frac{3}{2}], [\frac{3}{2}, 2], [2, \frac{5}{2}], [\frac{5}{2}, 3]$

$x_k = x_0 + k\Delta x$

left Riemann sum $= f(1)\Delta x + f(\frac{3}{2})\Delta x + f(2)\Delta x + f(\frac{5}{2})\Delta x$

$= 13.25$ ✓ is an over estimate

right Riemann sum $= f(\frac{3}{2})\Delta x + f(2)\Delta x + f(\frac{5}{2})\Delta x + f(3)\Delta x$

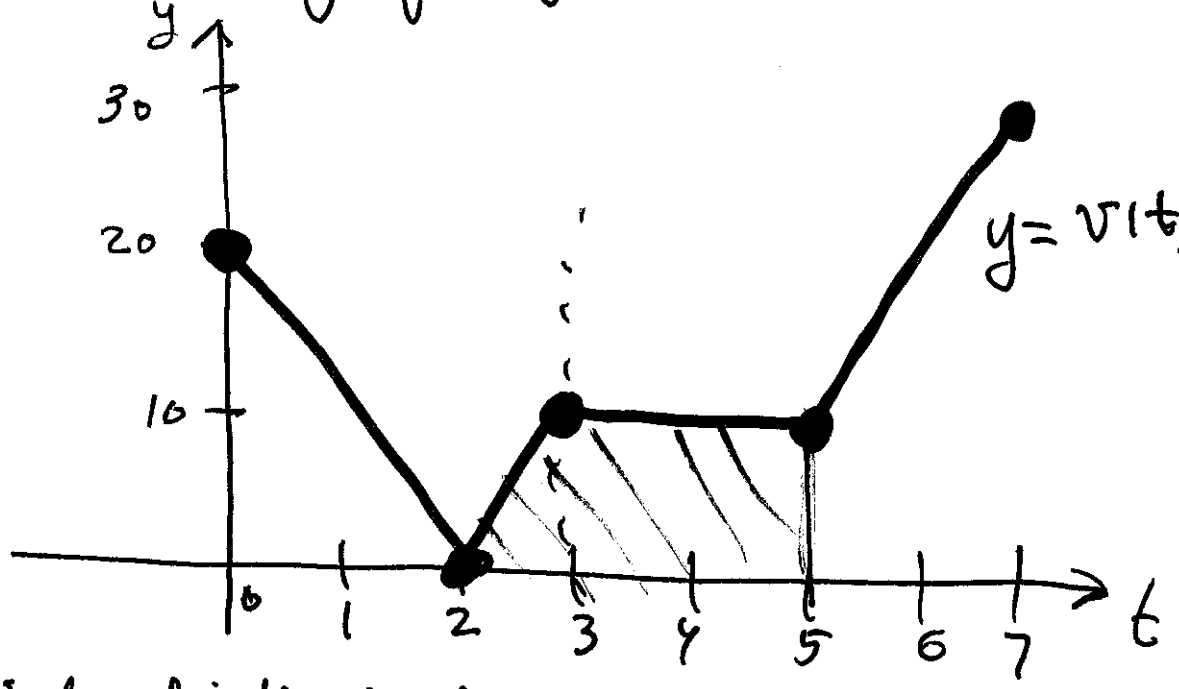
$= 9.25$ ✓ is an underestimate

midpt Riemann sum $= f\left(\frac{1+\frac{3}{2}}{2}\right)\Delta x + \bullet + \bullet + \bullet$

$= 11.375$ ✓

Exact area $= \frac{34}{3} = 11.333\ldots$

Ex2 Velocity of object



Find object's displacement from

$t=0$ to $t=3$:

$$D = \frac{1}{2}(2)(20) + \frac{1}{2}(1)(10) = 25 \text{ ft}$$

$t=2$ to $t=5$:

$$D = \frac{1}{2}(3+2)(10) = 25 \text{ ft.}$$