Lesson 31

§5.2 Definite Integrals

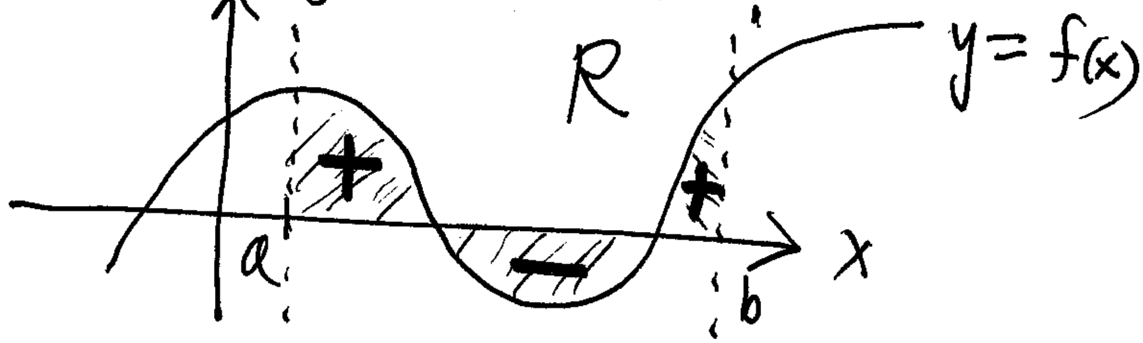
Sigma Notation: $\sum_{k=1}^{3} \frac{2k}{k+1} = \frac{2(1)}{2} + \frac{2(2)}{3} + \frac{2(3)}{4}$

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2} \quad \text{(Gauss)}$$

$$1 - \frac{1}{2} + \frac{1}{3} - \cdots + \frac{(-1)^{n+1}}{n} = \sum_{k=1}^{n} \frac{(-1)^{k+1}}{k}$$

$$f(x_1^*)\Delta x + \cdots + f(x_n^*)\Delta x = \sum_{k=1}^{n} f(x_k^*)\Delta x$$

Def: The net area of region R bounded by $y = f(x)$, x-axis, $x=a$, $x=b$ is sum of areas of R above x-axis minus sum of areas below.



$y = f(x)$, $[x_0, x_1], \ldots, [x_{n-1}, x_n]$
an arbitrary partition of $[a,b]$ and
$\Delta x_k = x_k - x_{k-1}$, let x_k^* be any pt in $[x_{k-1}, x_k]$
the <u>general Riemann sum of f</u>

$$\sum_{k=1}^{n} f(x_k^*)\Delta x_k$$

. Let $\Delta = \max\{\Delta_1, \Delta_2, \ldots, \Delta_n\}$

<u>Def</u>: $f(x)$ is <u>integrable</u> on $[a,b]$ if over all partitions of $[a,b]$ and all possible x_k^*

$$\lim_{\Delta \to 0} \sum_{k=1}^{n} f(x_k^*)\Delta x_k$$

is a unique number.

Moreover,

$$\lim_{\Delta \to 0} \sum_{k=1}^{n} f(x_k^*)\Delta x_k = \boxed{\int_a^b f(x)\,dx}$$

<u>Definite Integral</u>
of over $[a,b]$

Recall,

'Functions' $\rightarrow$ $\underbrace{\int f(x)\,dx}$

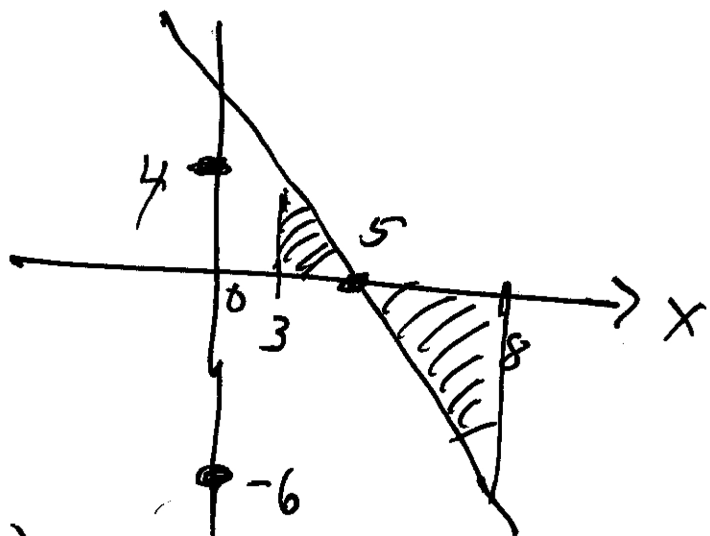
Indefinite Integral of f
(i.e. antiderivative of f)

$\boxed{\int_a^b f(x)\,dx}$ $\leftarrow$ number

Definite integral of f
(not area under curve)

$\boxed{\text{Ex 1}}$ Using geometry, evaluate

ⓐ $\int_3^8 \underbrace{(10-2x)}_{f(x)}\,dx$

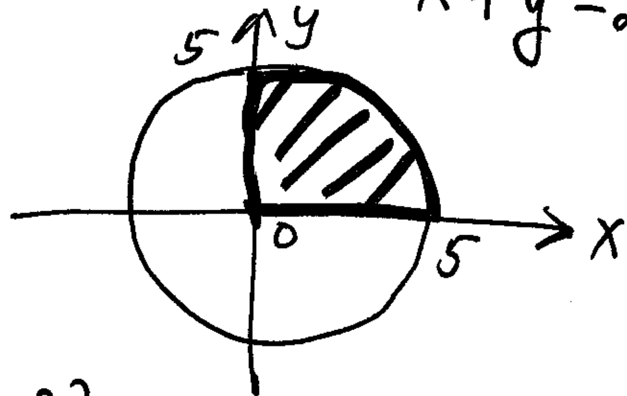


$= \frac{1}{2}(2)(4) - \frac{1}{2}(3)(6) = -5$ ✓

(b) $\int_0^5 \underbrace{\sqrt{25-x^2}}_{y=f(x)}\, dx$

$y=\sqrt{25-x^2}$

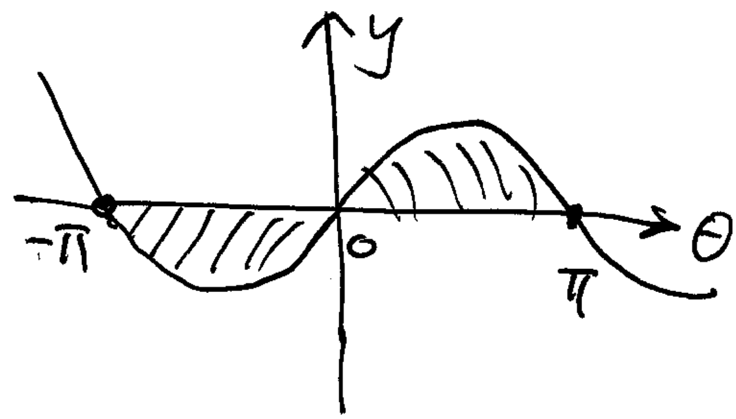
$x^2+y^2=25$



$= \frac{1}{4}\{\pi(5)^2\}$

(c) $\int_{-\pi}^{\pi} \sin\theta \, d\theta$

$= 0$



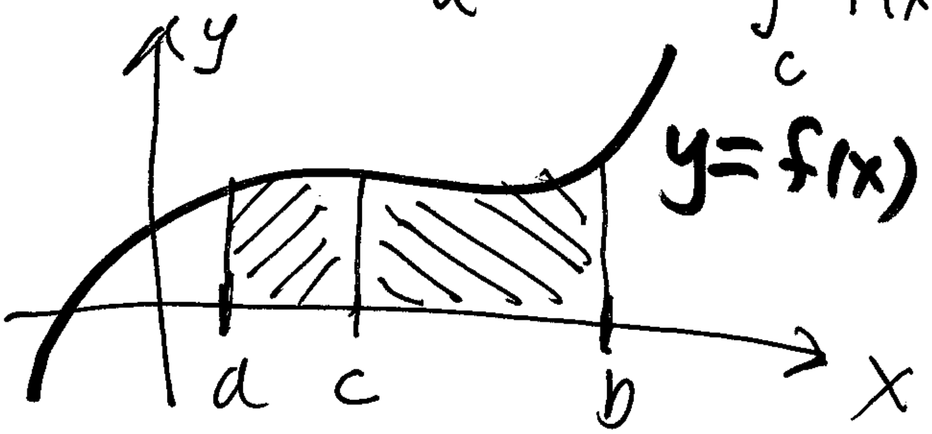
Basic Properties

1. $$\int_b^a f(x)dx = -\int_a^b f(x)dx$$

2. $$\int_a^a f(x)dx = 0$$

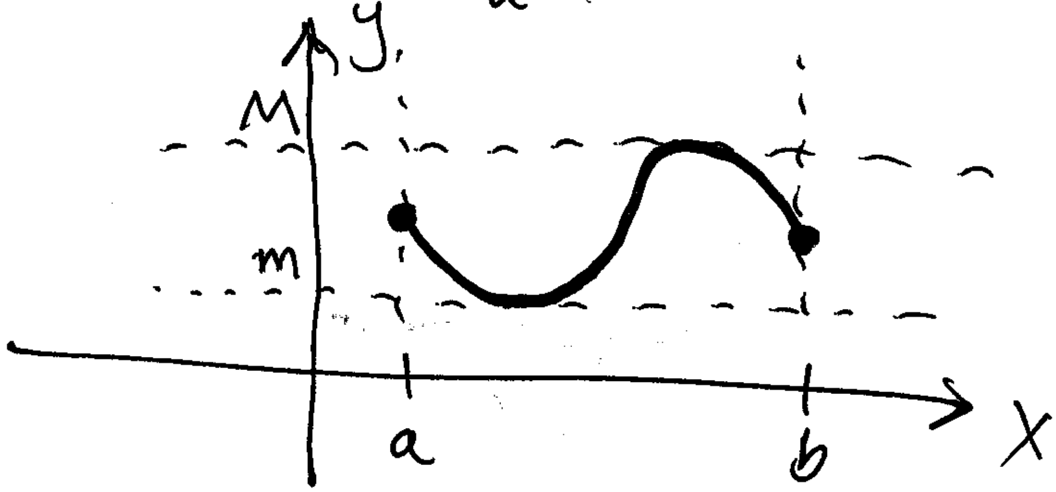
3. $$\int_a^b \{kf(x)+g(x)\}dx = k\int_a^b f(x)dx + \int_a^b g(x)dx$$

4. If $a \leq c \leq b$ then
$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$



5. If $m \le f(x) \le M$ on $[a,b]$

then $m(b-a) \le \int_a^b f(x)\,dx \le M(b-a)$



$\boxed{\text{Ex2}}$ Find upper & lower bounds

$$\int_1^3 (x^3 - 12x + 20)\,dx$$

find abs min & Max
$f(x) = x^3 - 12x + 20$
over $[1, 3]$

$m = 4$, $M = 11$ use Property 5