Lesson 4

§2.3 — Computing Limits

Limit Laws: ~~Suppose~~ $\lim_{x\to a} f(x) = L$, $\lim_{x\to a} g(x) = M$ and $n>0$.

1. $\lim_{x\to a}\{f(x)+g(x)\} = \lim_{x\to a} f(x) + \lim_{x\to a} g(x) = L+M$
2. $\lim_{x\to a}\{f(x)-g(x)\} = \lim_{x\to a} f(x) - \lim_{x\to a} g(x) = L-M$
3. $\lim_{x\to a}\{c f(x)\} = c \lim_{x\to a} f(x) = cL$
4. $\lim_{x\to a} f(x)g(x) = \left(\lim_{x\to a} f(x)\right)\left(\lim_{x\to a} g(x)\right) = LM$
5. $\lim_{x\to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)} = \dfrac{L}{M}$, provided $M \neq 0$
6. $\lim_{x\to a}\{f(x)\}^n = \{\lim_{x\to a} f(x)\}^n = L^n$
7. $\lim_{x\to a}\{f(x)^{\frac{1}{n}}\} = \{\lim_{x\to a} f(x)\}^{\frac{1}{n}} = L^{\frac{1}{n}}$ when n is even, need $f(x)>0$ for x near a

If $\lim_{x\to 1} f(x) = \frac{1}{2}$, $\lim_{x\to 1} g(x) = -64$

find $$\lim_{x\to 1} \frac{\sqrt[3]{g(x)}}{f(x)} = \frac{\lim_{x\to 1}\{g(x)^{\frac{1}{3}}\}}{\lim_{x\to 1} f(x)} = \frac{\{\lim_{x\to 1} g(x)\}^{1/3}}{\lim_{x\to 1} f(x)}$$

$$= \frac{(-64)^{1/3}}{1/2} = -8$$

Polynomials:

$$\lim_{x\to a} p(x) = \lim_{x\to a}\{c_n x^n + c_{n-1}x^{n-1} + \cdots + c_0\}$$
$$= c_n a^n + c_{n-1}a^{n-1} + \cdots + c_0$$
$$= p(a)$$

Hence

Polynomials: $\lim_{x\to a} p(x) = p(a)$

Rational Functions: $\lim_{x\to a} \frac{p(x)}{q(x)} = \frac{p(a)}{q(a)}$; $q(a) \neq 0$

Ex 1 Find limits (if they exist):

ⓐ $\lim_{x\to 2} \frac{3x^2+1}{x+8} = \frac{3(2^2)+1}{2+8} = \frac{13}{10}$

ⓑ $\lim_{x\to -1} \frac{(x-1)^2-4}{5x+5} = \lim_{x\to -1} \frac{x^2-2x-3}{5x+5}$

$= \lim_{x\to -1} \frac{\cancel{(x+1)}(x-3)}{5\cancel{(x+1)}} = \lim_{x\to -1} \frac{x-3}{5} = \frac{-4}{5}$

ⓒ $\lim_{x\to 9} \left(\frac{\sqrt{x}-3}{x-9}\right)$ <u>Recall</u>: $a^2-b^2 = (a+b)(a-b)$

$= \lim_{x\to 9} \left(\frac{\sqrt{x}-3}{x-9}\right)\left(\frac{\sqrt{x}+3}{\sqrt{x}+3}\right)$

$= \lim_{x\to 9} \frac{\cancel{(x-9)}}{\cancel{(x-9)}(\sqrt{x}+3)} = \frac{1}{\sqrt{9}+3} = \frac{1}{6}$

One-sided limits

$$\lim_{x \to a^+} \{f(x)\}^{\frac{1}{n}} = \left\{ \lim_{x \to a^+} f(x) \right\}^{\frac{1}{n}}$$

(when n is even need $f(x) \geq 0$ ~~for~~ all x near a with $x > a$)

Similarly $\lim_{x \to a^-} \{f(x)\}^{\frac{1}{n}}$

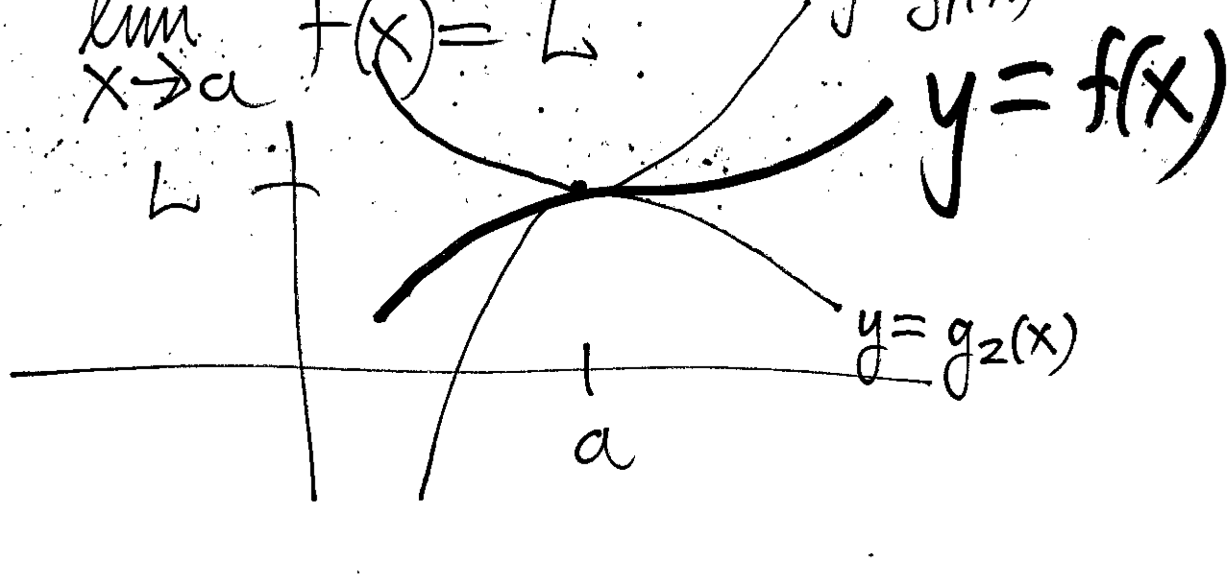
$\lim_{x \to 1^-} \sqrt{1-x} = 0$, $\lim_{x \to 1^+} \sqrt{1-x}$ DNE

Squeeze/Sandwich Theorem:

If $g_1(x) \leq f(x) \leq g_2(x)$ for all x near a (but not equal to a)

and if $\lim_{x \to a} g_1(x) = L$ and if $\lim_{x \to a} g_2(x) = L$

then $\lim_{x \to a} f(x) = L$



For eg given $6+\sqrt{x+3} \le f(x) \le x^2-2x+9$

for all x near 1

compute $\underbrace{\lim_{x\to 1} f(x)}_{=8}$

$8 \qquad = 8 \qquad 8$