Lesson 4

§2.3 – Computing Limits

Limit Laws: Suppose $\lim_{x\to a} f(x) = L$, $\lim_{x\to a} g(x) = M$

suppose $n > 0$

① $\lim_{x\to a}\{f(x)+g(x)\} = \lim_{x\to a} f(x) + \lim_{x\to a} g(x) = L+M$

② $\lim_{x\to a}\{f(x)-g(x)\} = \lim_{x\to a} f(x) - \lim_{x\to a} g(x) = L-M$

③ $\lim_{x\to a}\{cf(x)\} = c\lim_{x\to a} f(x) = cL$

④ $\lim_{x\to a}\{f(x)g(x)\} = \{\lim_{x\to a} f(x)\}\{\lim_{x\to a} g(x)\} = LM$

⑤ $\lim_{x\to a}\left\{\frac{f(x)}{g(x)}\right\} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)} = \frac{L}{M}$ provided $M \neq 0$

⑥ $\lim_{x\to a}\{f(x)^n\} = \{\lim_{x\to a} f(x)\}^n = L^n$

⑦ $\lim_{x\to a}\{f(x)^{\frac{1}{n}}\} = \{\lim_{x\to a} f(x)\}^{\frac{1}{n}} = L^{\frac{1}{n}}$ when n is even $f(x) > 0$ for all x near a

If $\lim\limits_{x \to 1} f(x) = \frac{1}{2}$ and $\lim\limits_{x \to 1} g(x) = -64$

find $\lim\limits_{x \to 1} \dfrac{\sqrt[3]{g(x)}}{f(x)} = \dfrac{\lim\limits_{x \to 1} \{g(x)\}^{1/3}}{\lim\limits_{x \to 1} f(x)}$

$$= \frac{\left\{\lim\limits_{x \to 1} g(x)\right\}^{1/3}}{\lim\limits_{x \to 1} f(x)} = \frac{(-64)^{1/3}}{1/2} = -8$$

<u>Polynomials:</u> $\lim\limits_{x \to a} p(x) = \lim\limits_{x \to a} \{c_n x^n + c_{n-1} x^{n-1} + \cdots + c_0\}$

$$= c_n a^n + c_{n-1} a^{n-1} + \cdots + c_0$$

$$= p(a)$$

Hence,

<u>Polynomials:</u> $\lim\limits_{x \to a} p(x) = p(a)$

<u>Rational functions:</u> $\lim\limits_{x \to a} \dfrac{P(x)}{q(x)} = \dfrac{P(a)}{q(a)}$ provided $q(a) \neq 0$

$\boxed{\text{Ex1}}$ Find limits (if they exist)

ⓐ $\lim_{x\to 2} \dfrac{3x^2+1}{x+8} = \dfrac{3(2)^2+1}{2+8} = \dfrac{13}{10}$

ⓑ $\lim_{x\to -1} \dfrac{(x-1)^2-4}{5x+5} = \lim_{x\to -1} \dfrac{x^2-2x-3}{5x+5}$

$= \lim_{x\to -1} \dfrac{\cancel{(x+1)}(x-3)}{5\cancel{(x+1)}} = \lim_{x\to -1} \dfrac{x-3}{5} = \dfrac{-4}{5}$

ⓒ $\lim_{x\to 9} \left(\dfrac{\sqrt{x}-3}{x-9}\right)$ <u>Recall:</u> $a^2-b^2=(a-b)(a+b)$

$= \lim_{x\to 9} \left(\dfrac{\sqrt{x}-3}{x-9}\right)\left(\dfrac{\sqrt{x}+3}{\sqrt{x}+3}\right)$

$= \lim_{x\to 9} \dfrac{\cancel{(x-9)}}{\cancel{(x-9)}(\sqrt{x}+3)} = \dfrac{1}{\sqrt{9}+3} = \dfrac{1}{6}$

One-sided limits:

$$\lim_{x\to a^+}\{f(x)\}^{\frac{1}{n}} = \left\{\lim_{x\to a^+} f(x)\right\}^{\frac{1}{n}}$$

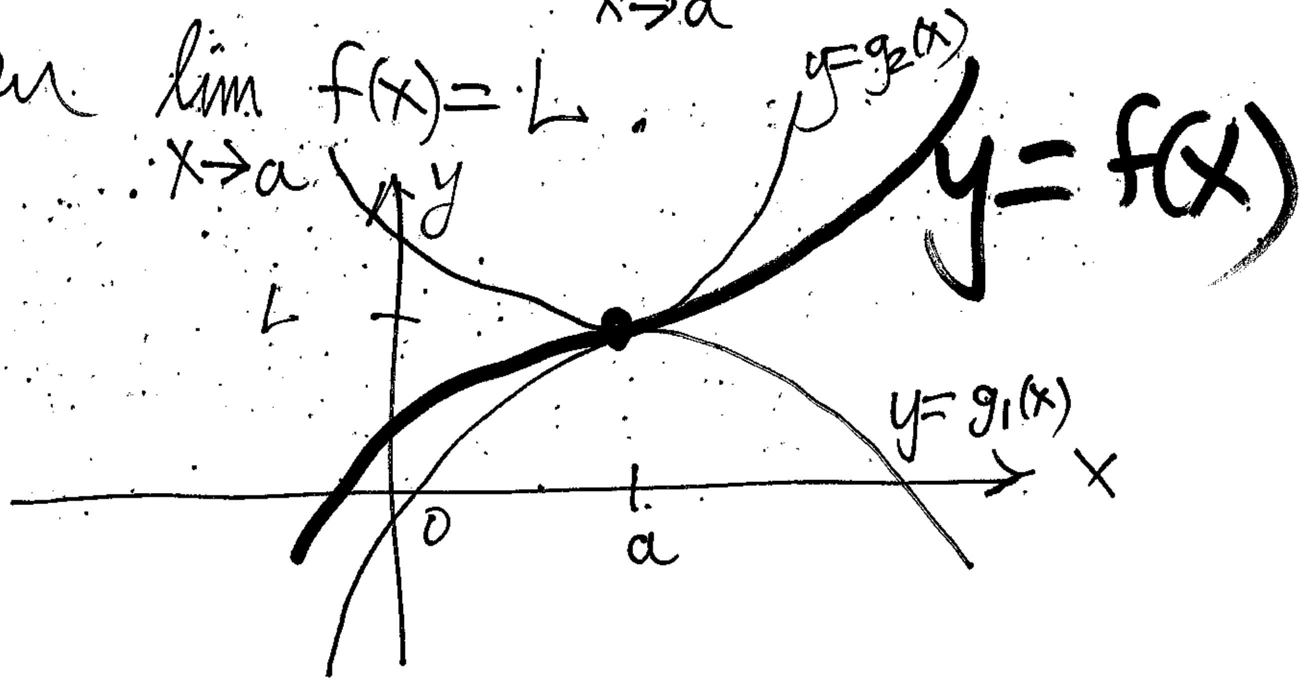
when n is even, $f(x)\geq 0$ for all $x>a$ near a

Similarly $x\to a^-$
$(x<a)$

$\lim_{x\to 1^-}\sqrt{1-x} = 0$ but $\lim_{x\to 1^+}\sqrt{1-x}$ DNE

Squeeze/Sandwich Theorem:

If $g_1(x)\leq f(x)\leq g_2(x)$ for all x near a
(but not equal to a)
and if $\lim_{x\to a} g_1(x) = L$ and $\lim_{x\to a} g_2(x) = L$
then $\lim_{x\to a} f(x) = L$.



$$6+\sqrt{x+3} \le f(x) \le x^2-2x+9$$

all x near 1

$$\lim_{x\to 1} f(x) = ?$$

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