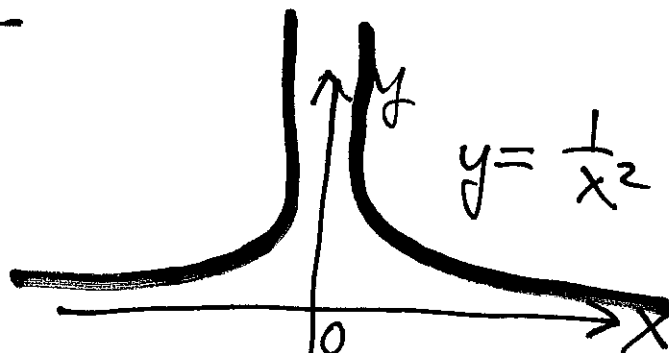


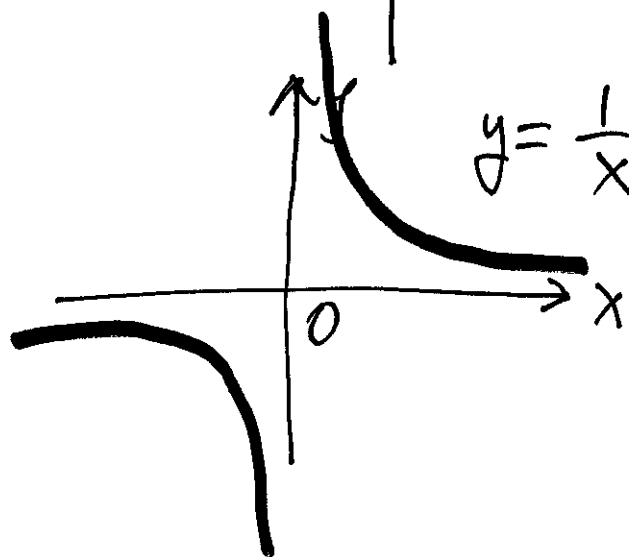
§2.4 - Infinite Limits

✓ $\lim_{x \rightarrow 0} \frac{1}{x^2} = \infty$

Also $\lim_{x \rightarrow \infty} \frac{1}{x^2} = 0$



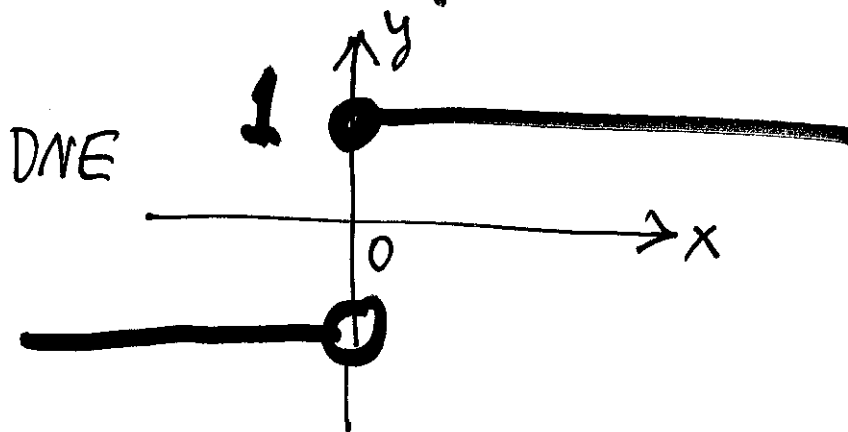
✓ $\lim_{x \rightarrow 0} \frac{1}{x}$



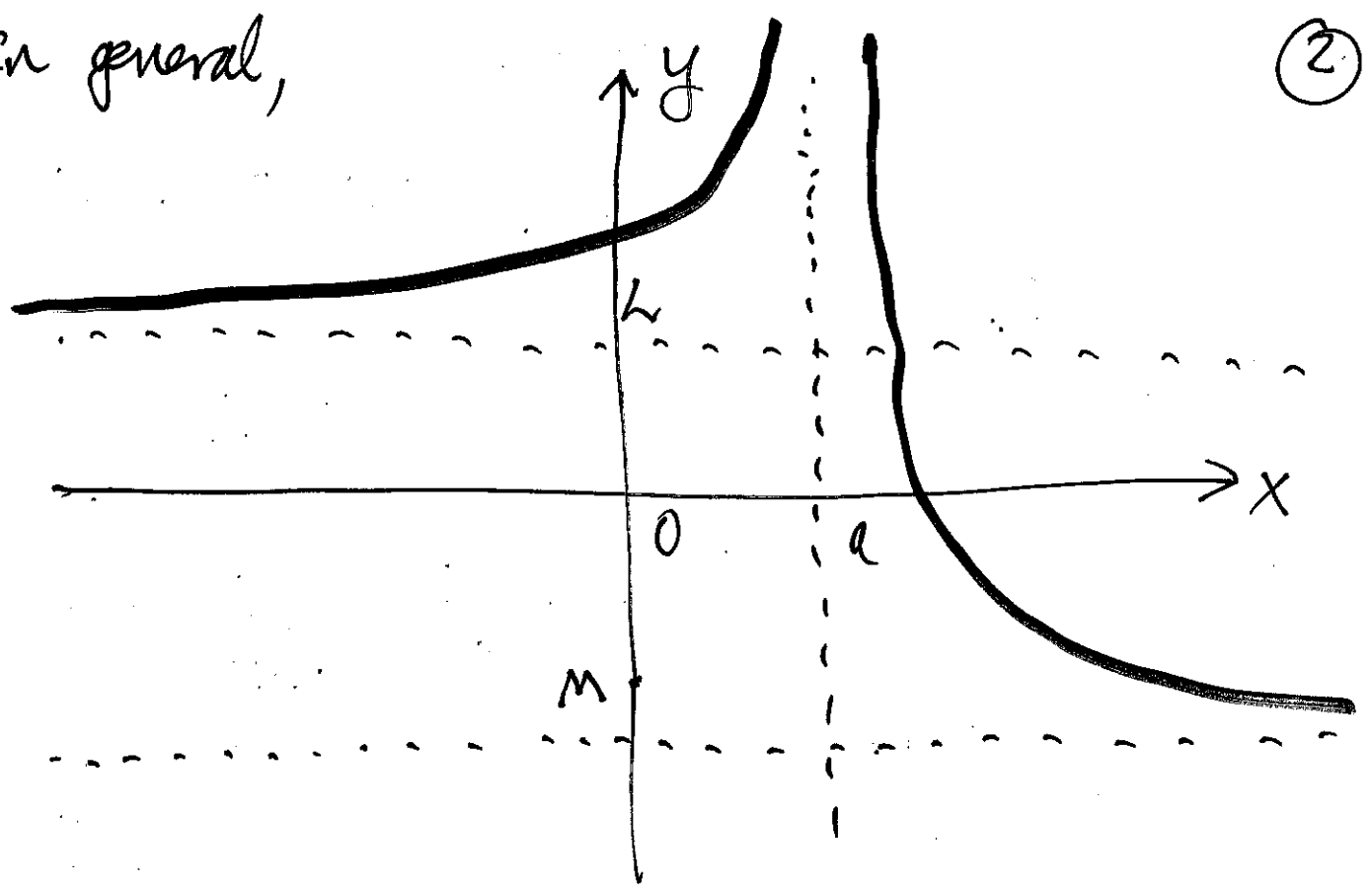
$\lim_{x \rightarrow 0^+} \frac{1}{x} = \infty$

$\lim_{x \rightarrow 0^-} \frac{1}{x} = -\infty$

✓ $\lim_{x \rightarrow 0} \frac{|x|}{x}$



In general,



Def: Suppose f is defined for all x near a (not necessarily at a)

① $\lim_{x \rightarrow a} f(x) = \infty$ means $f(x)$ grows arbitrarily large for all x suff. close to a (not equal to a) on both sides of a

② $\lim_{x \rightarrow a} f(x) = -\infty$ means $f(x) < 0$ and its magnitude grows arb. large for all x suff. close to a ($\neq a$) on both sides

These limits don't exist because L is infinite

Remark: A limit DNE $\left\{ \begin{array}{l} f \text{ not defined near } a \text{ (3)} \\ \text{values } f(x) \text{ don't converge} \\ \text{if limit is infinite} \end{array} \right.$

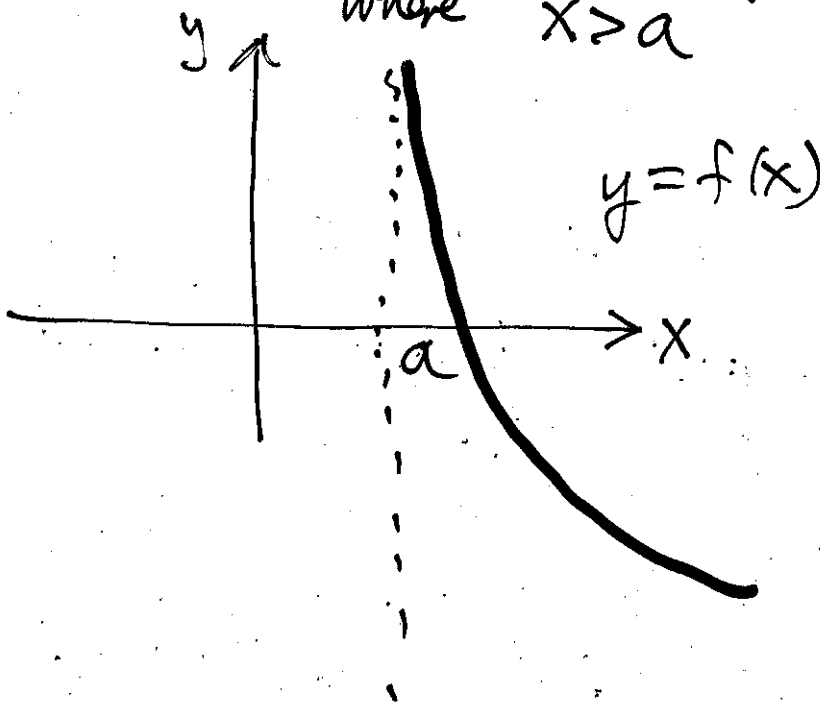
Ex 1 Analyze

$$\lim_{x \rightarrow 2} \frac{3x}{(x^2-4)^2}, \quad \lim_{x \rightarrow -2} \frac{3x}{(x^2-4)^2}$$

One-sided Infinite Limits

$$\lim_{x \rightarrow a^+} f(x) = \infty$$

means f grows arb. large
for all x suff. close to a
where $x > a$



Def: If any of these 3 limits

$$\lim_{x \rightarrow a} f(x), \lim_{x \rightarrow a^+} f(x), \lim_{x \rightarrow a^-} f(x)$$

is $+\infty$ or $-\infty$, then $x=a$ is a Vertical Asymptote of $f(x)$.

Remark: For rational functions $f(x) = \frac{P(x)}{Q(x)}$ the vertical asymp. are among zeros of $Q(x)$

Ex2 Find all vertical asymptotes:

a) $f(x) = \frac{4(x-2)}{(x-1)(x+3)} \Rightarrow x=1$
 $x=-3$ possible vert. asymp.

$\lim_{x \rightarrow 1^+} f(x) = \frac{1}{(x-1)} \left[\frac{4(x-2)}{(x+3)} \right] = -\infty$ ($x > 1$)

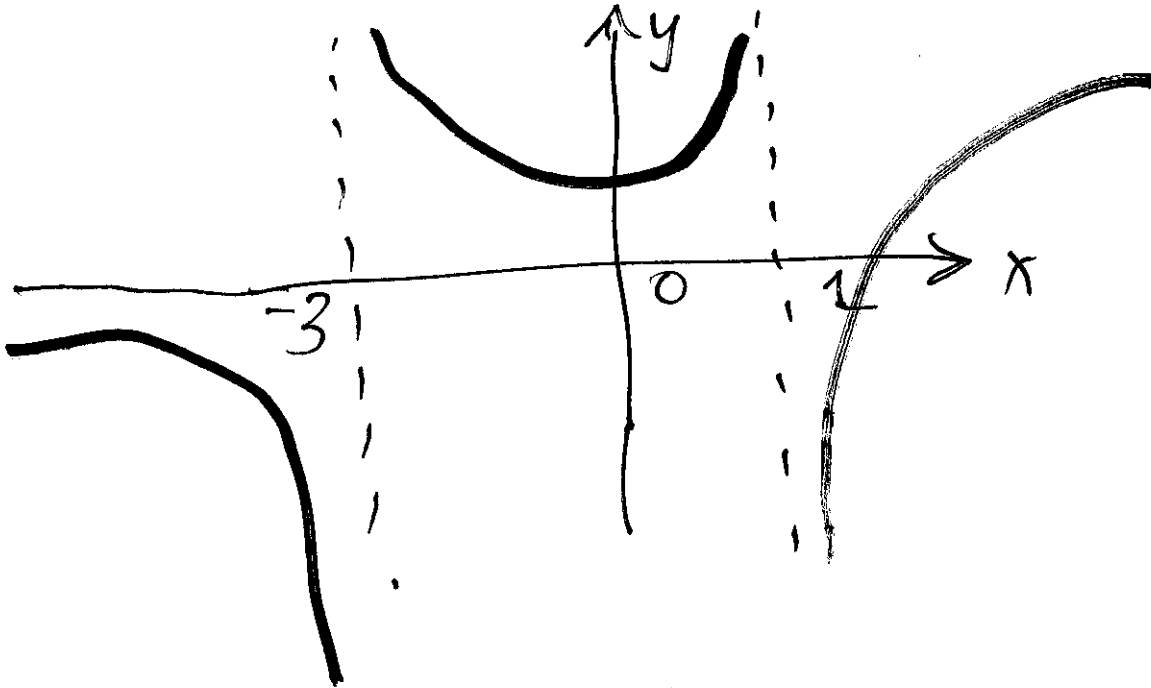
$x=1$ Vert. Asymp

$\lim_{x \rightarrow -3^+} f(x) = \frac{1}{(x+3)} \left[\frac{4(x-2)}{(x-1)} \right]$

$x > -3$

$= +\infty$

$x=-3$ Vert. Asymp



(b)
$$f(x) = \frac{4x^2 - 8x}{x(x^2 - 4)(x - 3)^2} = \frac{4x(x - 2)}{x(x - 2)(x + 2)(x - 3)^2}$$

Possible vert. asymp. $x = \cancel{0}, \cancel{2}, -2, 3$

Away from these pts $f(x) = \frac{4}{(x + 2)(x - 3)^2}$

$\lim_{x \rightarrow 0} f(x) = \frac{4}{18}$

$\lim_{x \rightarrow -2^-} f(x) = \lim_{x \rightarrow -2^-} \underbrace{\frac{1}{(x + 2)}}_{< 0} \left[\underbrace{\frac{4}{(x - 3)^2}}_{> 0} \right]$ $x < -2$

$= -\infty$

$x = -2$ Vert. Asymp

Also $x = 3$