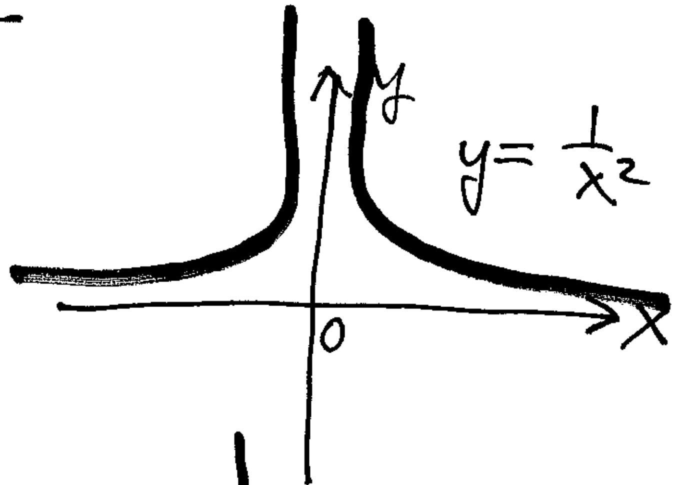
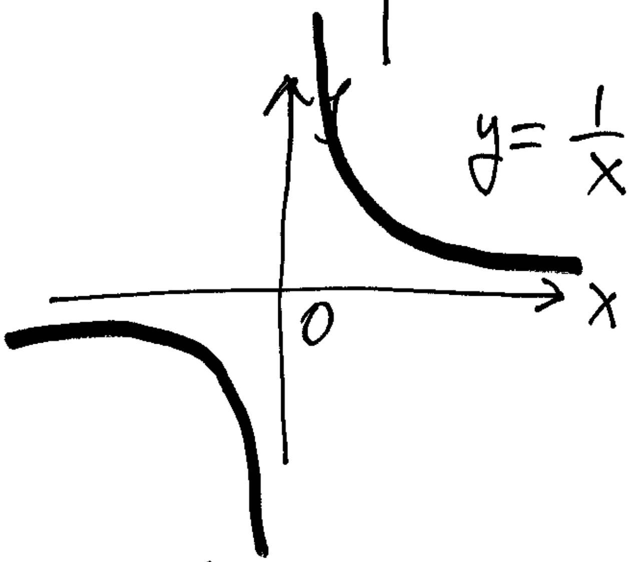
§2.4 - Infinite Limits

✓ $\lim\limits_{x\to 0} \dfrac{1}{x^2} = \infty$



Also $\lim\limits_{x\to \infty} \dfrac{1}{x^2} = 0$

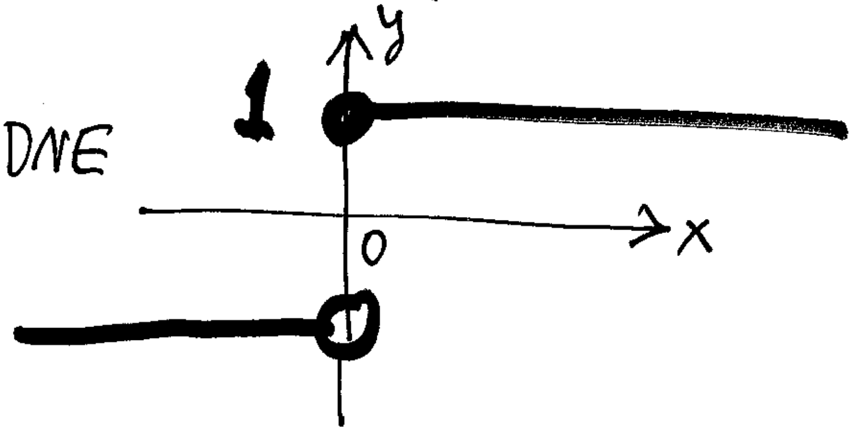
✓ $\lim\limits_{x\to 0} \dfrac{1}{x}$



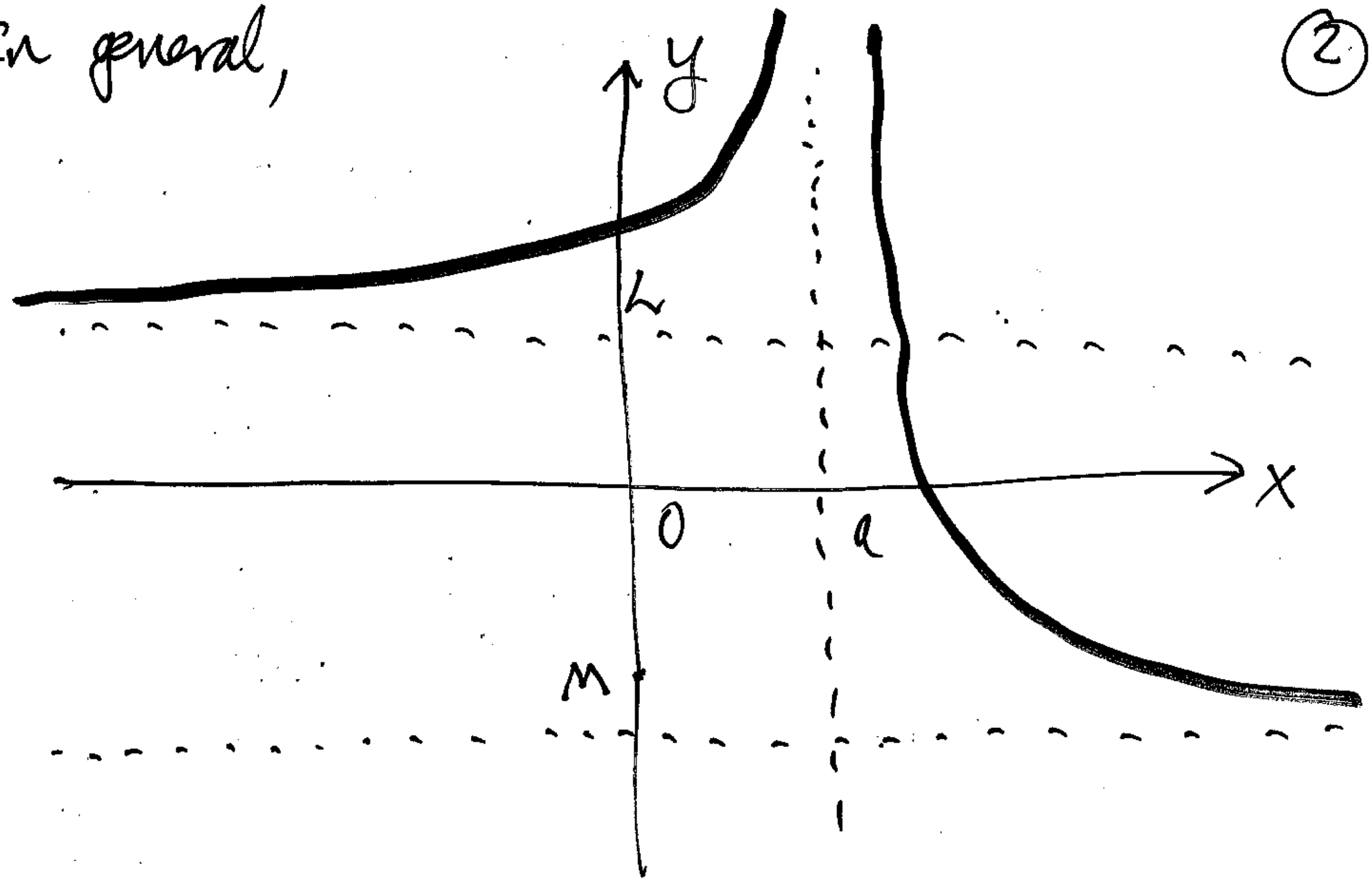
$\lim\limits_{x\to 0^+} \dfrac{1}{x} = \infty$

$\lim\limits_{x\to 0^-} \dfrac{1}{x} = -\infty$

✓ $\lim\limits_{x\to 0} \dfrac{|x|}{x}$ DNE



In general,



Def: Suppose f is defined for all x near a (not necessarily at a)

1. $\boxed{\lim_{x\to a} f(x) = \infty}$ means f(x) grows arbitrarily large for all x suff. close to a (not equal to a) on both sides of a

2. $\boxed{\lim_{x\to a} f(x) = -\infty}$ means $f(x) < 0$ and its magnitude grows arb. large for all x suff. close to a ($\neq a$) on both sides

These limits don't exist because L is infinite

Remark: A limit DNE

- f not defined near a
- values f(x) don't converge
- if limit is infinite

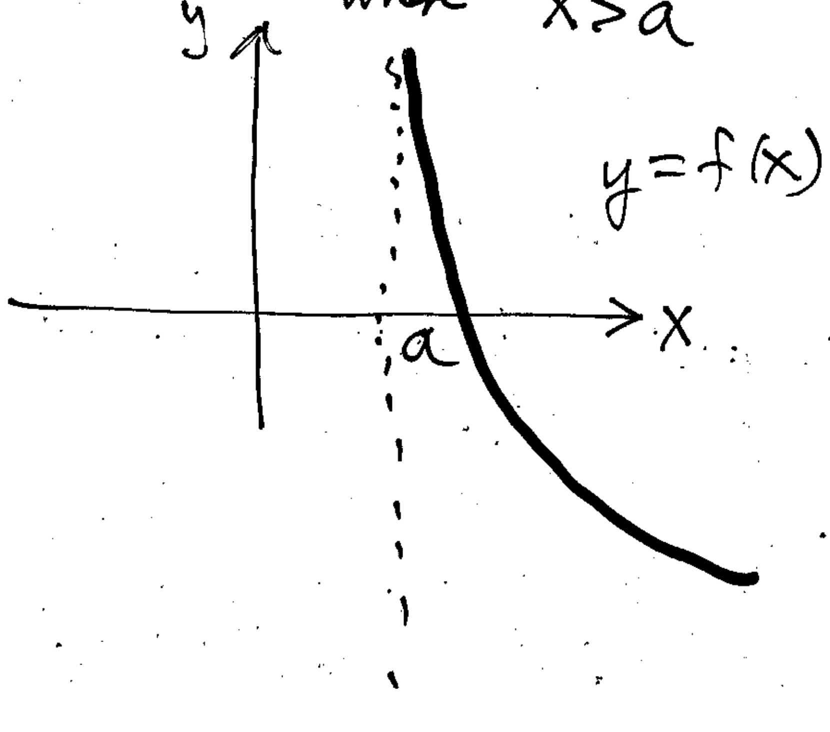
$\boxed{\text{Ex1}}$ Analyze

$$\lim_{x \to 2} \frac{3x}{(x^2-4)^2}, \quad \lim_{x \to -2} \frac{3x}{(x^2-4)^2}$$

One-sided Infinite Limits

$$\boxed{\lim_{x \to a^+} f(x) = \infty}$$

means f grows arb. large for all ~~x~~ suff. close to a where $x > a$



Def: If any of these 3 limits

$$\lim_{x\to a} f(x), \lim_{x\to a^+} f(x), \lim_{x\to a^-} f(x)$$

is $+\infty$ or $-\infty$, then $\boxed{x=a}$ is a Vertical Asymptote of $f(x)$.

Remark: For rational functions $f(x)=\dfrac{p(x)}{q(x)}$ the vertical asymp. are among zeros of $q(x)$

$\boxed{\text{Ex2}}$ Find all vertical asymptotes:

ⓐ $f(x)=\dfrac{4(x-2)}{(x-1)(x+3)} \Rightarrow x=1, x=-3$ possible vert. asymp.

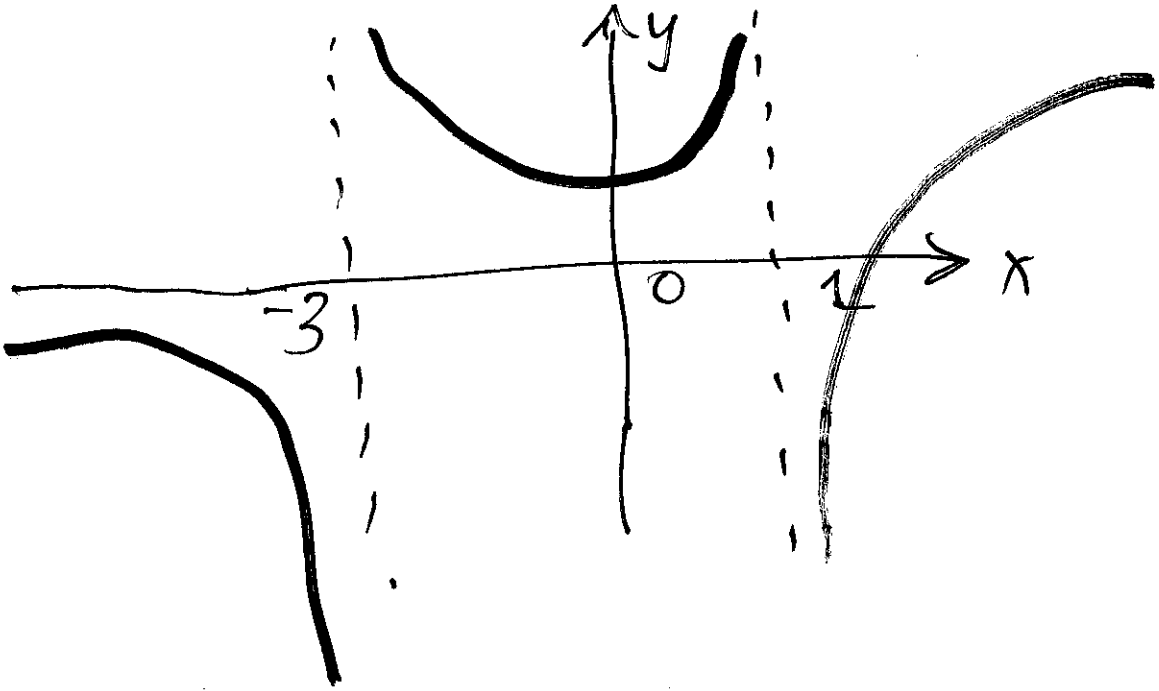
$$\lim_{x\to 1^+} f(x)=\underset{>0}{\frac{1}{(x-1)}}\Big[\underset{<0}{\frac{4(x-2)}{(x+3)}}\Big]=-\infty \quad x>1$$

$\therefore \boxed{x=1}$ Vert. Asymp

$$\lim_{x\to -3^+} f(x)=\lim \underset{>0}{\frac{1}{(x+3)}}\Big[\underset{>0}{\frac{4(x-2)}{(x-1)}}\Big] \quad x>-3$$

$$=+\infty$$

$\boxed{x=-3}$ Vert. Asymp



(b) $f(x) = \dfrac{4x^2-8x}{x(x^2-4)(x-3)^2} = \dfrac{4x(x-2)}{x(x-2)(x+2)(x-3)^2}$

Possible vert. asymp. $x = \cancel{0}, \cancel{2}, -2, 3$

Away from these pts $f(x) = \dfrac{4}{(x+2)(x-3)^2}$

$\lim_{x\to 0} f(x) = \frac{4}{18}$

$\lim_{x\to -2^-} f(x) = \lim_{x\to -2^-} \underbrace{\frac{1}{(x+2)}}_{<0} \underbrace{\left[\frac{4}{(x-3)^2}\right]}_{>0}$ $x < -2$

$= -\infty$

$\boxed{x=-2}$ Vert. Asymp

Also $\boxed{x=3}$