Lesson 7

§2.6 Continuity

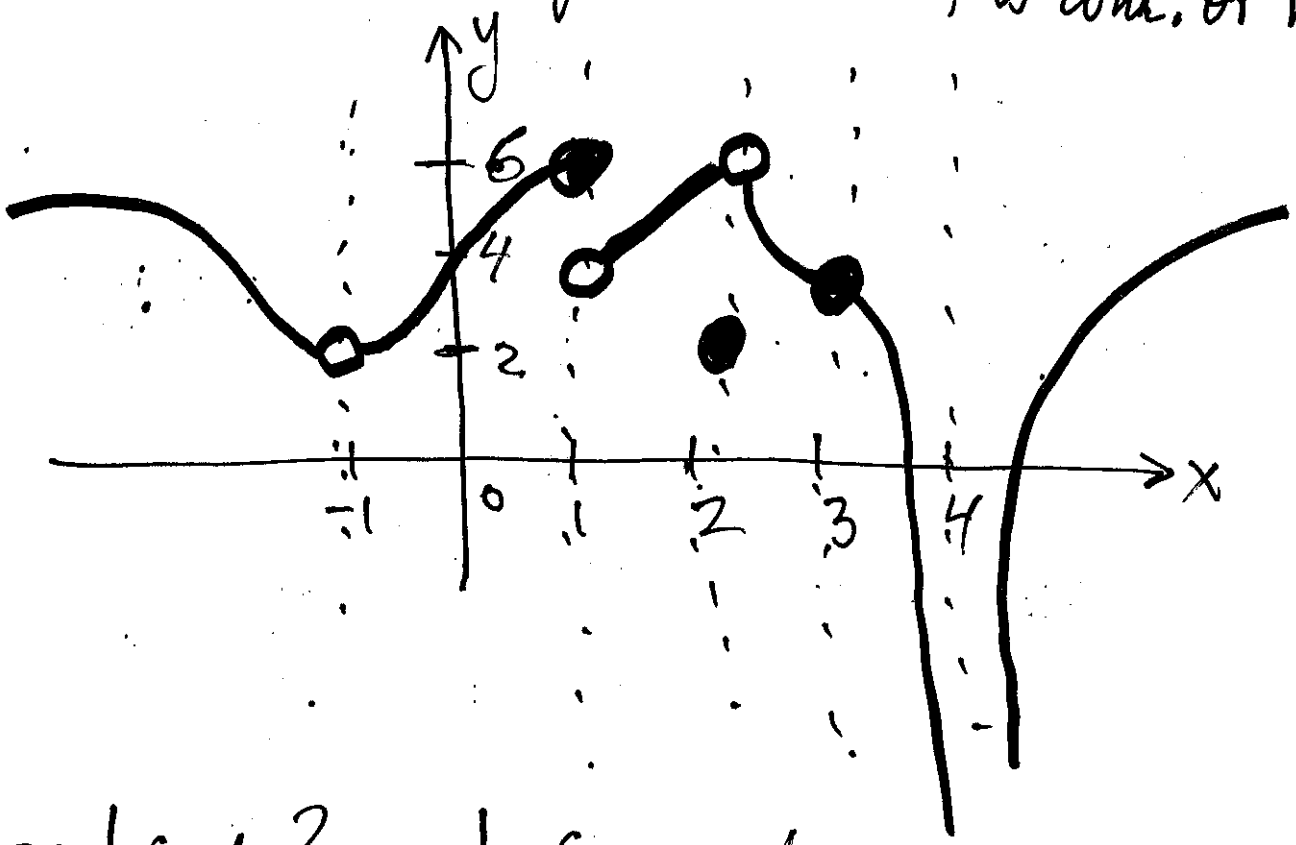
Def: f is continuous at a if $\lim_{x \to a} f(x) = f(a)$

Continuity Checklist: (f is cont. at a $\iff$)

1. $f(a)$ is defined
2. $\lim_{x \to a} f(x)$ exists (and is finite)
3. $\lim_{x \to a} f(x) = f(a)$

If f fails any of these at a then f is discontinuous at a

Ex 1 Given graph of f determine if points in the table are places where f is cont. or not:



x	Cont. ?	Comment
-1	NO; (1), (3) fail	removable discontinuity (if $f(-1)=2$ then cont.)
0	YES	
1	NO; (2), (3) fail	jump discontinuity
2	NO; (3) fails	removable discontinuity
3	YES	
4	NO; (1), (2), (3) fail	infinite discont.

Thm: If f, g are cont. at a then those are also cont. at a.

① $f \pm g$, fg, $\frac{f}{g}$ (provided $g(a) \neq 0$)

② $f(x)^n$ $(n = 1, 2, 3 \ldots)$

③ $f(x)^{\frac{1}{n}}$ (for n even $f(a) > 0$)

✓ Polynomials are cont for all x

✓ Rational functions $\frac{p(x)}{q(x)}$ is cont. for all x except $q(x) = 0$

$f(x) = \frac{(x-1)(x+1)}{x(x-1)}$ cont. for all $x \neq 0$, $x \neq 1$

($x = 1$ is a removable discontinuity)

Thm (Limits of Compositions)

① If g is cont at a and f cont at $g(a)$ then

$$\lim_{x \to a} f(g(x)) = f\left(\lim_{x \to a} g(x)\right) = f(g(a))$$

② If $\lim_{x \to a} g(x) = L$ and f is cont. at L then

$$\lim_{x \to a} f(g(x)) = f\left(\lim_{x \to a} g(x)\right) = f(L)$$

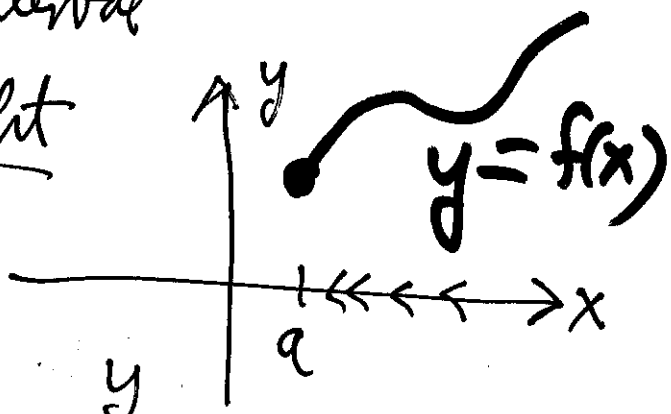
Ex $\lim_{x\to 2} \cos\left(\frac{\pi(x^2-4)}{3x-6}\right)$

$= \cos\left(\lim_{x\to 2} \frac{\pi(x-2)(x+2)}{3(x-2)}\right) = \cos\left(\frac{4\pi}{3}\right) = -\frac{1}{2}$

Remarks:

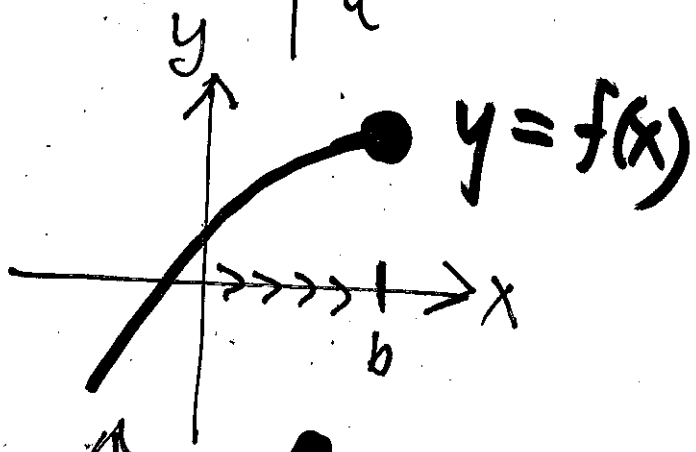
- f cont on (a,b) means f cont at each pt in interval
- f is continuous from right

$$\lim_{x\to a^+} f(x) = f(a)$$

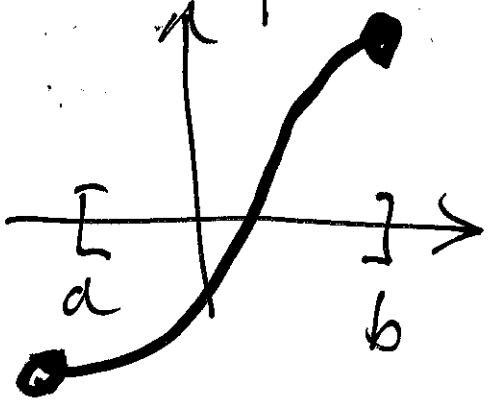


- f cont. from left

$$\lim_{x\to b^-} f(x) = f(b)$$



- f cont on $[a,b]$



$$f(x) = \frac{\sin^{-1} x}{\ln(2x)}$$ continuous?

trig, inv. trig
exp & log
are cont. on their domains.

$\sin^{-1} x$ cont. $-1 \le x \le 1$

$\ln(2x)$ cont. $2x > 0$

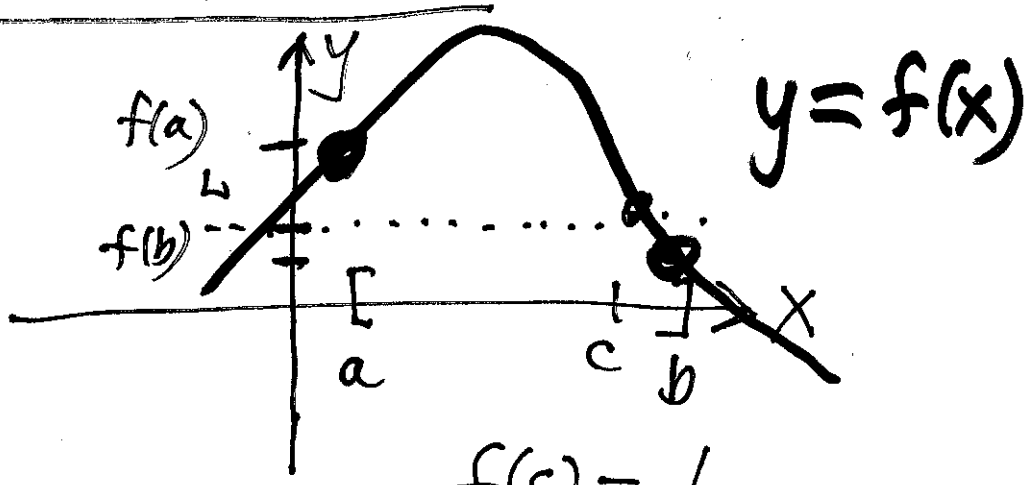
Need $\ln(2x) \neq 0$

$2x \neq 1$

$x \neq \frac{1}{2}$

$\therefore$ $(0, \frac{1}{2})$ and $(\frac{1}{2}, 1]$

Intermediate Value Thm:



$f(c) = L$