§2.6 Continuity

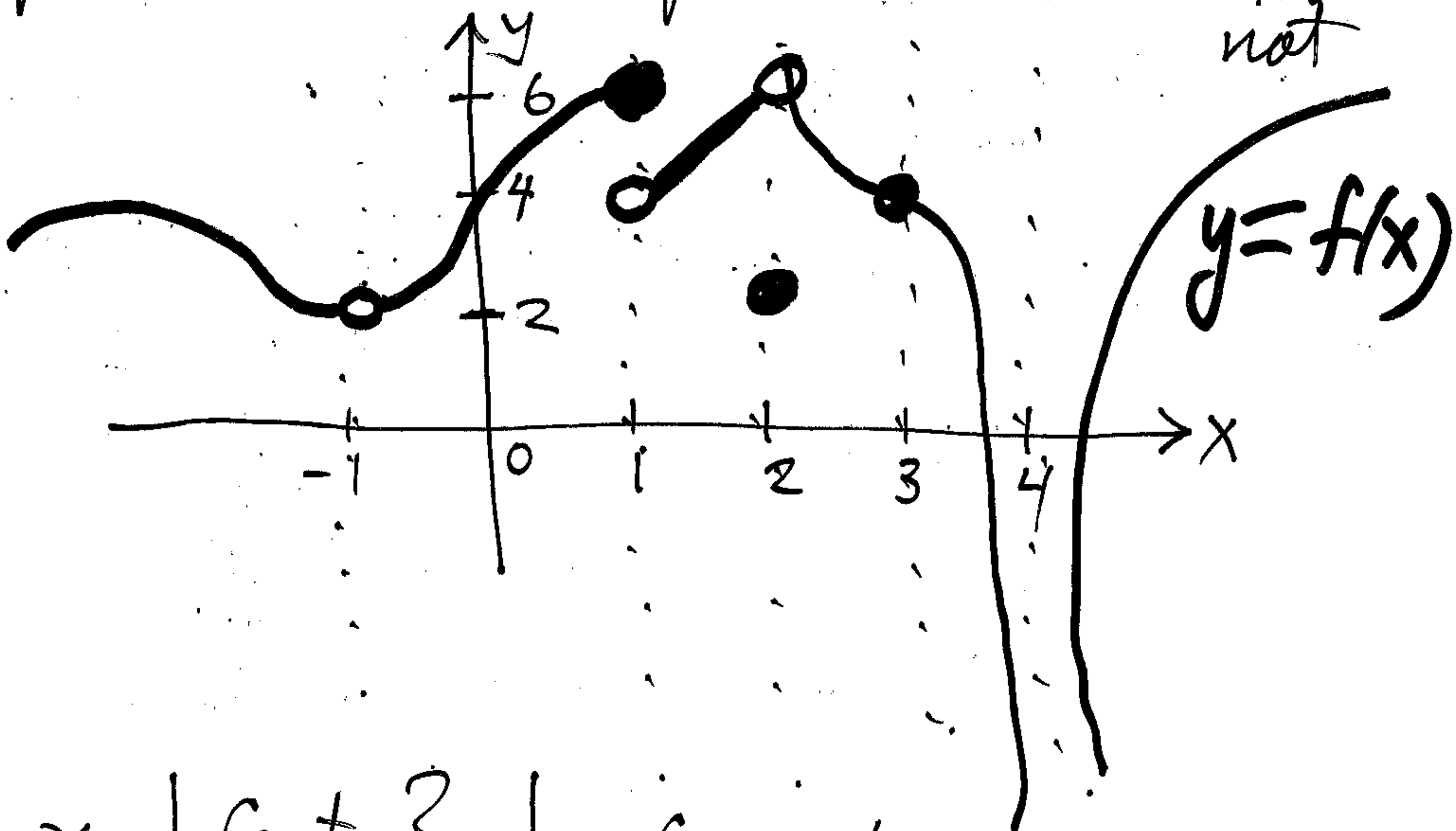
Def: f is continuous at a if $\boxed{\lim_{x \to a} f(x) = f(a)}$

Continuity Checklist: f is cont at a $\iff$

1. $f(a)$ is defined
2. $\lim_{x \to a} f(x)$ exists (and is finite)
3. $\lim_{x \to a} f(x) = f(a)$

If f fails any of these at a then f is discontinuous at a

$\boxed{\text{Ex 1}}$ Given graph of f determine if points in table are places where f is cont. or not



x	Cont.?	Comment
−1	No; (1), (3) fail	removable discontinuity (If $f(-1)=2$ then cont)
0	YES	
1	No; (2), (3) fail	jump discontinuity
2	NO; (3) fails	removable discontinuity
3	YES	
4	NO; (1) (2) (3) fails	infinite discont.

Thm: If f, g are cont. at a, then these are also cont at a

1. $f \pm g$, fg, $\frac{f}{g}$ (provided $g(a) \neq 0$)
2. $f(x)^n$ $(n = 1, 2, \cdots)$
3. $f(x)^{1/n}$ (for n even $f(a) > 0$)

* Polynomials, Rational Functions, trig, inv trig, exp, logs are cont. for all x in their domains

For example $f(x) = \frac{(x-1)(x+1)}{x(x-1)}$ domain is all x where $x \neq 0$, $x \neq 1$

✓ hence f cont $x \neq 0$, $x \neq 1$

Thm (Limits of Compositions)

1. If g is cont. at a and f is cont at g(a) then

$$\lim_{x \to a} f(g(x)) = f\left(\lim_{x \to a} g(x)\right) = f(g(a))$$

2. If $\lim_{x \to a} g(x) = L$ and f is cont at L then

$$\lim_{x \to a} f(g(x)) = f\left(\lim_{x \to a} g(x)\right) = f(L)$$

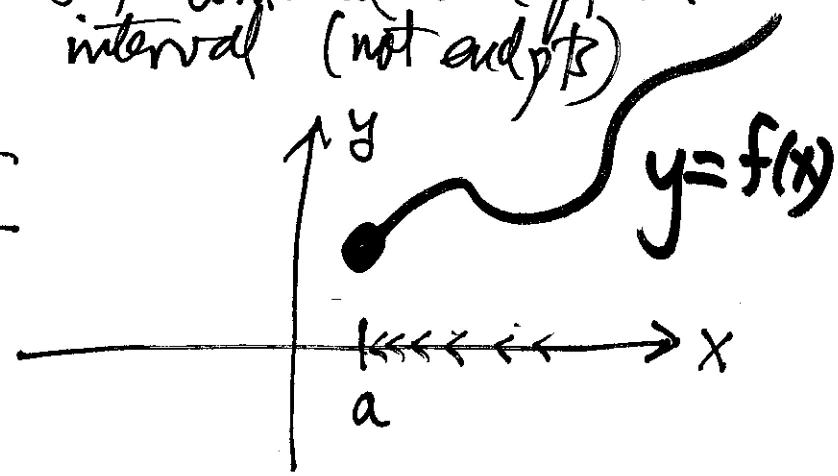
$\boxed{\text{Ex2}}$ $\displaystyle \lim_{x \to 2} \cos\left(\frac{\pi(x^2-4)}{3x-6}\right)$

$$= \cos\left(\lim_{x\to 2} \frac{\pi\cancel{(x-2)}(x+2)}{3\cancel{(x-2)}}\right) = \cos\frac{4\pi}{3} = -\frac{1}{2}$$

Remarks:

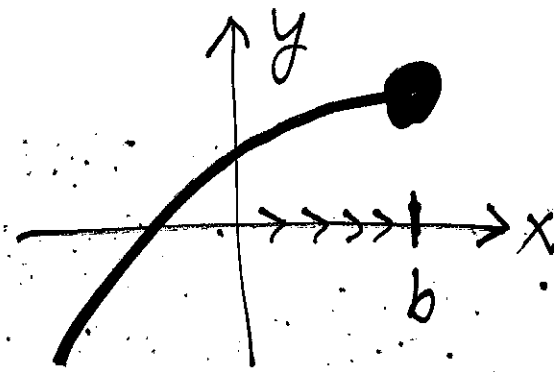
- f cont. on (a,b) means f cont. at each pt in interval (not end pts)
- f is cont. from right

$$\boxed{\lim_{x\to a^+} f(x) = f(a)}$$

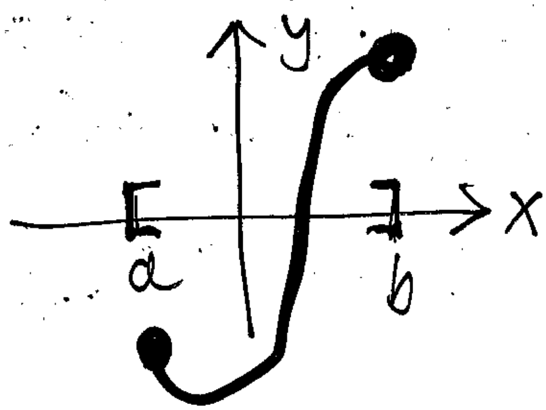


- f cont. from left

$$\boxed{\lim_{x\to b^-} f(x) = f(b)}$$



- f cont. on $[a,b]$



$f(x) = \dfrac{\sin^{-1} x}{\ln(2x)}$ cont. where?

$\sin^{-1} x$ cont $-1 \le x \le 1$

$\ln(2x)$ cont $2x > 0$ i.e. $x > 0$

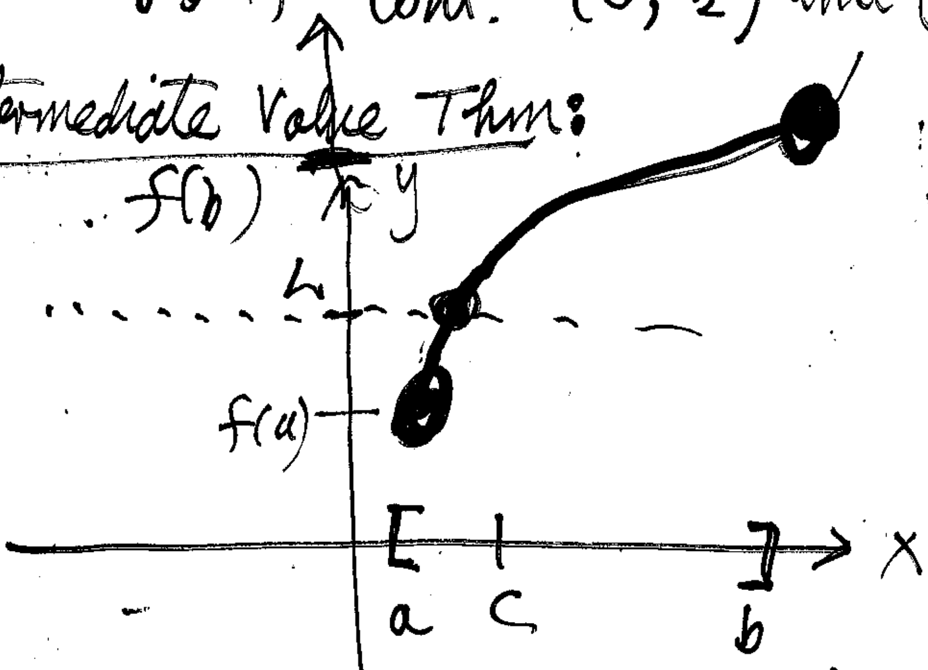
$\ln(2x) \neq 0$

$x \neq 1/2$

$2x \neq 1$

$\therefore$ f cont. $(0, \frac{1}{2})$ and $(\frac{1}{2}, 1]$

Intermediate Value Thm:



$f(c) = L$