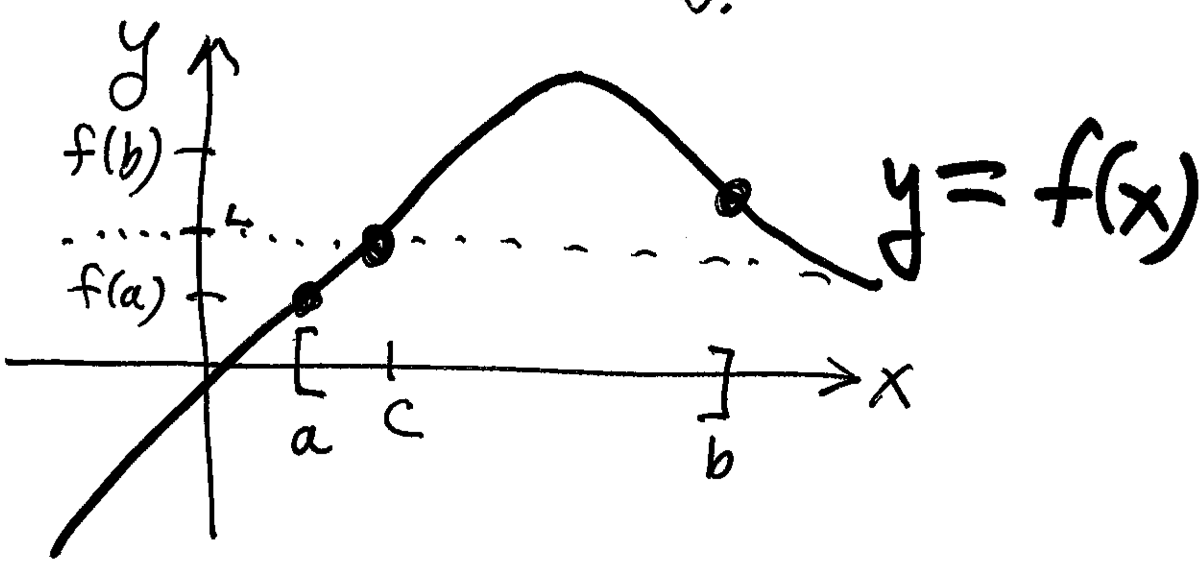
Last time

Intermediate Value Thm:

If f is cont. on $[a,b]$ and L is any number strictly between $f(a)$ and $f(b)$ there is at least one c such that $f(c) = L$ and $a < c < b$.



Ex Investing \$1000 at end of each year for 10 yrs with annual interest rate r gives

$$A(r) = 1000 \frac{[(1+r)^{10} - 1]}{r}$$

Is there a rate r in $[0.01, 0.10]$ for which $A = \$15,000$?

Soln: A cont. on [0.01, 0.10]

$A(0.01) = \$10,462$

$A(0.10) = \$15,937$

$L = 15,000$ between 2 values

$\therefore$ YES

Lesson 8

§3.1 – Introducing the derivative

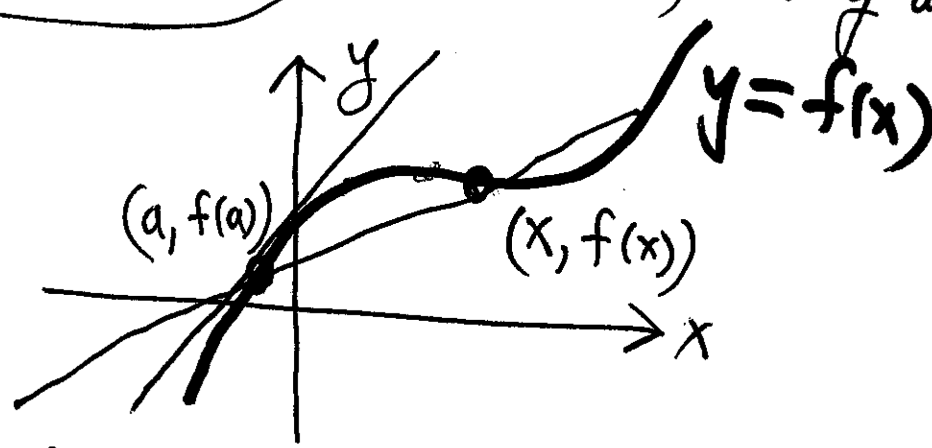
Recall

① If $y = s(t)$ = displacement of object

$$v_{av} = \frac{s(t) - s(a)}{t - a} \quad \text{average velocity over } [a, t]$$

$$v_{ins} = \lim_{t \to a} \frac{s(t) - s(a)}{t - a} \quad \text{(instantaneous) velocity at } a$$

② $y = f(x)$



$$m_{sec} = \frac{f(x) - f(a)}{x - a}$$

$$m_{tan} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

③ $y = g(x)$ then let $\Delta x = x - a$

$\Delta y = g(x) - g(a)$

average rate of change of $y = g(x)$ over $[a, x]$

$$\frac{\Delta y}{\Delta x} = \frac{g(x) - g(a)}{x - a}$$

(instantaneous) rate of change of $y = g(x)$ at a

$$\lim_{x \to a} \frac{\Delta y}{\Delta x} = \lim_{x \to a} \frac{g(x) - g(a)}{x - a}$$

Def: The derivative of f at a is

$$\boxed{f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}}$$

or equivalently

(if we let $h = x - a$)

$$\boxed{f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}}$$ ✓

If $f'(a)$ exists we say f is differentiable at a.

$\boxed{\text{Ex 1}}$ Find $f'(9)$ if $f(x) = 4\sqrt{x}$.
Find equation of the tangent line to graph at $x = 9$.

<u>Soln</u>: $f'(9) = \lim_{h \to 0} \dfrac{f(9+h) - f(9)}{h}$

$$= \lim_{h\to 0} \frac{4\sqrt{9+h} - 4\sqrt{9}}{h} = 4\lim_{h\to 0}\left(\frac{\sqrt{9+h}-3}{h}\right)$$

$$= 4\lim_{h\to 0}\left(\frac{\sqrt{9+h}-3}{h}\right)\left(\frac{\sqrt{9+h}+3}{\sqrt{9+h}+3}\right)$$

$$= 4\lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{9+h}+3)} = \frac{4}{\sqrt{9}+3} = \frac{4}{3+3} = \frac{2}{3}$$

Recall, equation of line through (x_0, y_0), slope m is

$$\boxed{y - y_0 = m(x - x_0)}$$

Tangent line $m = 2/3$, pt $= (9, f(9)) = (9, 12)$

$\therefore$ $y - 12 = \frac{2}{3}(x - 9)$ or $y = \frac{2}{3}x + 6$

$\boxed{\text{Ex 2}}$ If a ball thrown up at 96 ft/sec then its height is $s(t) = -16t^2 + 96t$. Find velocity at $t = 1$ sec and when it hits ground.

__Soln__: $s'(1) = \lim_{t \to 1} \dfrac{s(t) - s(1)}{t-1}$

$$= \lim_{t \to 1} \frac{[-16t^2 + 96t] - [80]}{t-1} = 64 \text{ ft/sec}$$

Hits ground $s(t) = 0$

$$-16t^2 + 96t = 0$$

$$16t(-t + 6) = 0 \qquad t = 0, \quad t = 6$$

__Check__ $s'(6) = \lim_{h \to 0} \dfrac{s(6+h) - s(6)}{h} = -96$ ft/sec

Ex3 For what k is f cont at $x=1$?

$$f(x)=\begin{cases} 2x^2, & \text{if } x\le 1 \\ \frac{k}{2}x-2, & \text{if } x>1 \end{cases}$$

For that k, is f differentiable at $x=1$?