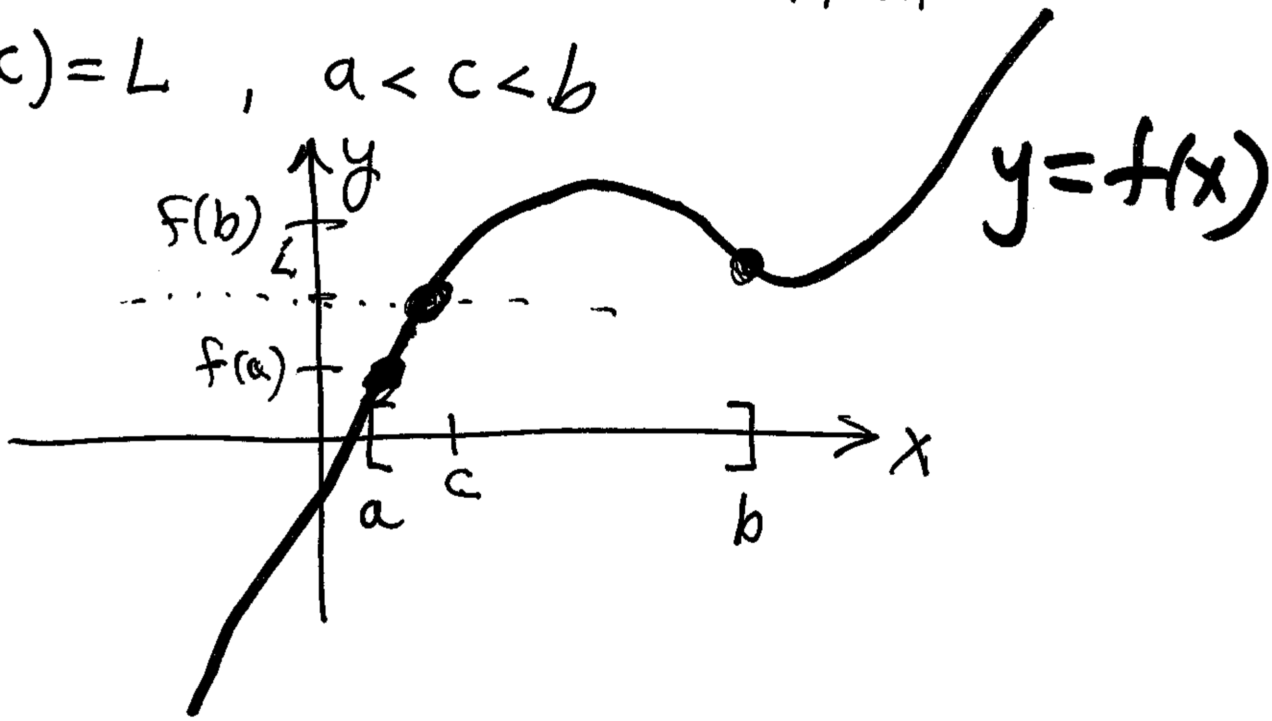
## Last time

Intermediate Value Thm: If $f$ is cont. on $[a, b]$ and $L$ any number strickly between $f(a), f(b)$ then there is a number $c$ such that

$$f(c) = L \quad , \quad a < c < b$$




$\boxed{\text{Ex}}$ Investing \$1000 at end of each year for 10 yrs with annual interest rate $r$ gives

$$A(r) = 1000 \frac{[(1+r)^{10} - 1]}{r} \text{ dollars}$$

Is there a rate $r$ in $[0.01, 0.10]$ for which $A = \$15,000$ ?

Soln: A cont. in [0.01, 0.10]

$A(0.01) = \$10,462$

$A(0.10) = \$15,937$

$L = \$15,000$ between 2 values

YES

# Lesson 8

## §3.1 – Introducing the derivative

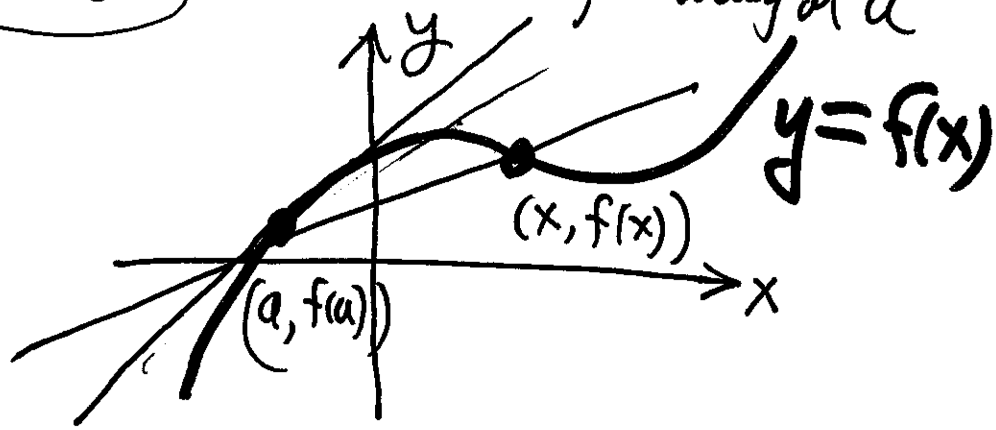
Recalling

① If $y = s(t) =$ displacement of object

$$v_{av} = \frac{s(t) - s(a)}{t - a}$$ average velocity over $[a, t]$

$$v_{ins} = \lim_{t \to a} \frac{s(t) - s(a)}{t - a}$$ (instantaneous) velocity at a

② $y = f(x)$




$$m_{sec} = \frac{f(x) - f(a)}{x - a}$$

$$m_{tan} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

③ $y = g(x)$ let $\Delta x = x - a$

$$\Delta y = g(x) - g(a)$$

average rate of change of g over $[a, x]$

$$\frac{\Delta y}{\Delta x} = \frac{g(x) - g(a)}{x - a}$$

(instantaneous) rate of change of g at a

$$\lim_{x \to a} \frac{\Delta y}{\Delta x} = \lim_{x \to a} \frac{g(x) - g(a)}{x - a}$$

**Def:** The *derivative* of f at a is

$$\boxed{f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}}$$

or equivalently (if $h = x - a$)

$$\boxed{f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}}$$ ✓

If $f'(a)$ exists then f is *differentiable* at a

**Ex1** Find $f'(9)$ if $f(x) = 4\sqrt{x}$

Find equation of tangent line to graph at $x=9$.

<u>Soln:</u> $$f'(9) = \lim_{h\to 0} \frac{f(9+h) - f(9)}{h}$$

$$= \lim_{h\to 0} \frac{4\sqrt{9+h} - 4\sqrt{9}}{h}$$

$$= 4 \lim_{h\to 0} \left(\frac{\sqrt{9+h} - 3}{h}\right)\left(\frac{\sqrt{9+h} + 3}{\sqrt{9+h} + 3}\right)$$

$$= 4 \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{9+h} + 3)} = 4 \frac{1}{(\sqrt{9}+3)} = \frac{4}{3+3} = \frac{2}{3}$$

Recall, <u>equation of line through $(x_0, y_0)$, slope $m$</u>

$$\boxed{y - y_0 = m(x - x_0)}$$

Tangent line $m = 2/3$, $x_0 = 9$, $y_0 = f(9) = 12$

$\therefore$ $\underline{y - 12 = 2/3\,(x-9)}$ or $\underline{y = \frac{2}{3}x + 6}$

$\boxed{\text{Ex 2}}$ If a ball is thrown up at 96 ft/sec then its height is $s(t) = -16t^2 + 96t$.
Find velocity at $t = 1$ sec and when it hits ground.



Soln: $s'(1) = \lim_{t \to 1} \dfrac{s(t) - s(1)}{t-1}$

$$= \lim_{t \to 1} \frac{[-16t^2 + 96t] - [80]}{t-1} = 64 \text{ ft/sec}$$

Hits ground $s(t) = 0$

$$-16t^2 + 96t = 0$$

$$16t(-t+6) \qquad t = 0$$

$$t = 6$$

You Check $s'(6) = -96$ ft/sec.