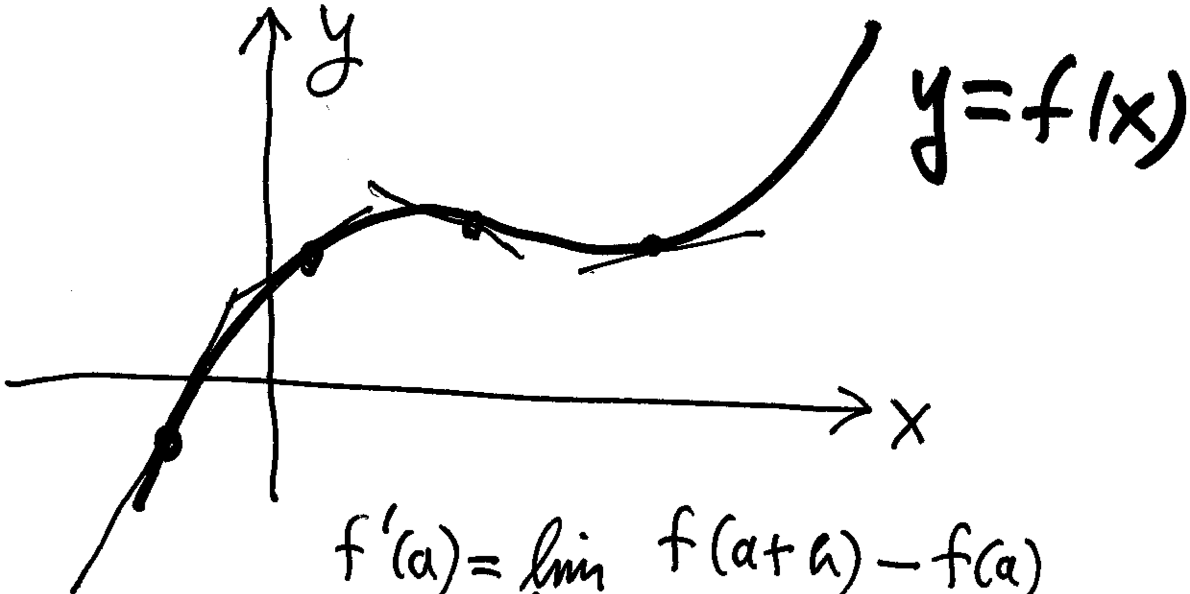
# How to be Unsuccessful

1. I stopped attending class.
2. I neither read the book nor took notes.
3. I never learned the material.
4. I am a JIT student.
5. I thought I was smart.

Lesson 9

## §3.2 – The derivative as a function




$$f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$

(or $f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$ )

**Def:** The derivative of $f$ is the function

$$\boxed{f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}}$$

provided limit exists and $x$ in domain of $f$.
We say $f$ is differentiable at $x$.
(else $f$ is not differentiable at $x$)

$\boxed{\text{Ex 1}}$ Compute $f'(x)$ if $f(x) = 4x + \frac{1}{\sqrt{x}}$

Soln: $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$$= \lim_{h\to 0} \frac{\left[4(x+h) + \frac{1}{\sqrt{x+h}}\right] - \left[4x + \frac{1}{\sqrt{x}}\right]}{h}$$

$$= \lim_{h\to 0} \frac{4h + \left(\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}\right)}{h}$$

$$= \underbrace{\lim_{h\to 0} \frac{4\cancel{h}}{\cancel{h}}}_{\text{provided exists}} + \underbrace{\lim_{h\to 0} \frac{\left(\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}\right)}{h}}_{\text{exists}}$$

provided exists and exists

$$\therefore \lim_{h\to 0} \frac{(\sqrt{x} - \sqrt{x+h})}{h\sqrt{x+h}\,\sqrt{x}} \left(\frac{\sqrt{x} + \sqrt{x+h}}{\sqrt{x} + \sqrt{x+h}}\right)$$

$$= \lim_{h\to 0} \frac{-\cancel{h}}{\cancel{h}\sqrt{x+h}\,\sqrt{x}\,(\sqrt{x} + \sqrt{x+h})}$$

$$= \frac{-1}{(\sqrt{x})(\sqrt{x})(2\sqrt{x})} = -\frac{1}{2x\sqrt{x}}$$

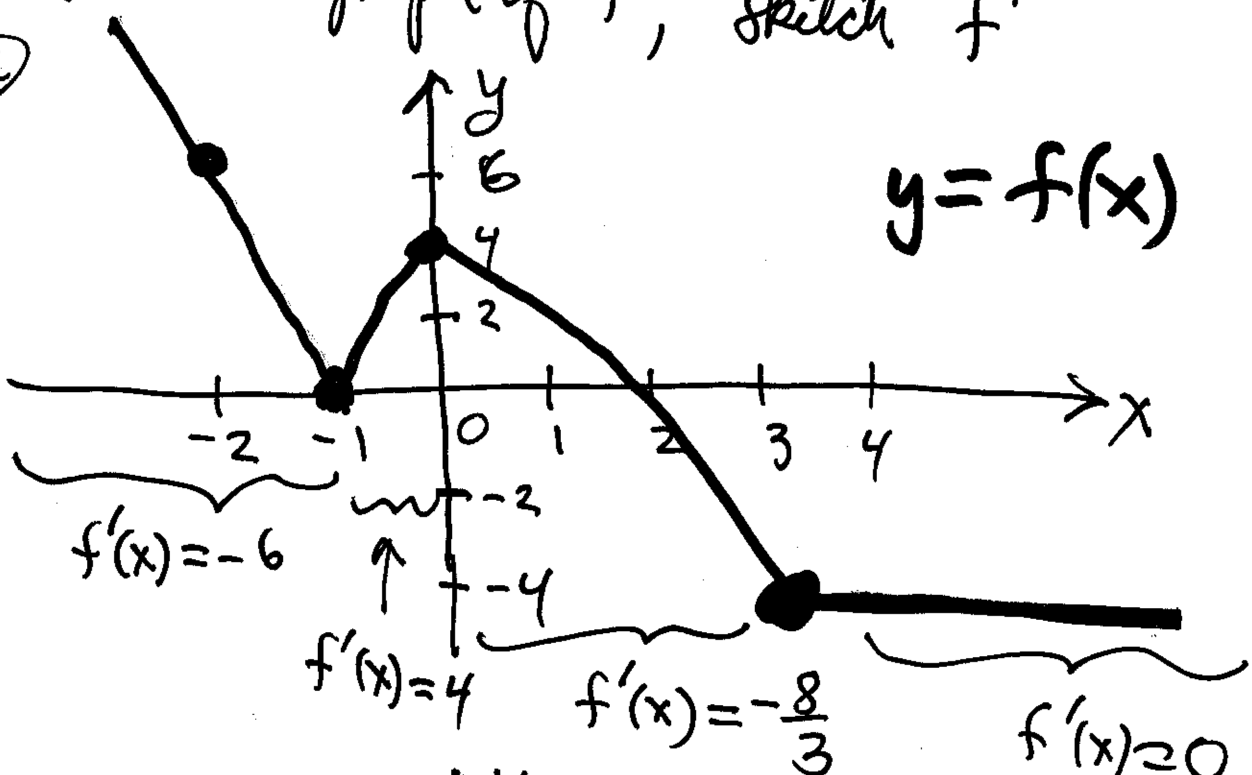
$$\therefore \; f'(x) = 4 - \frac{1}{2x\sqrt{x}} = 4 - \frac{1}{2x^{3/2}}$$

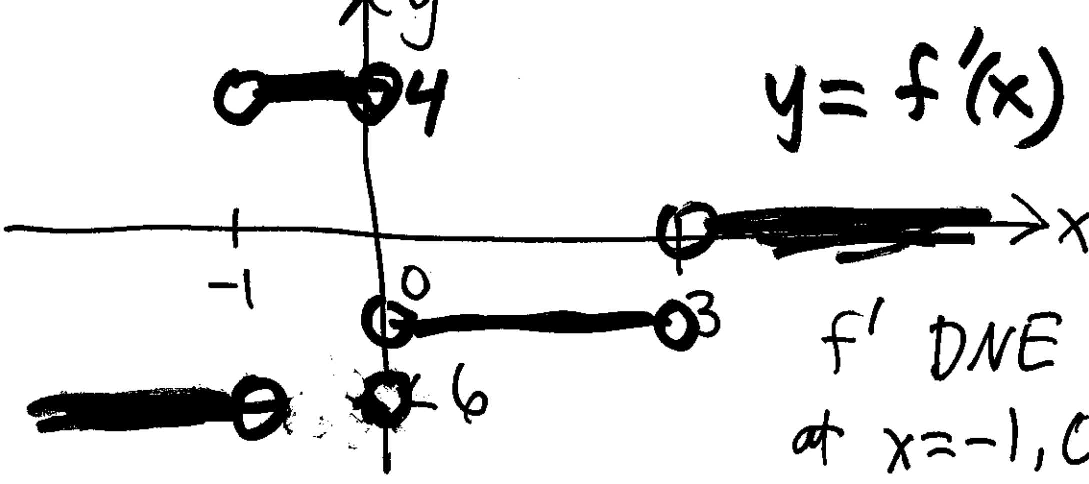
Notation: $y = f(x)$ then the derivative:

$$f'(x), \; \frac{df}{dx}, \; y', \; \frac{dy}{dx}$$

$\boxed{\text{Ex2}}$ Given graph of $f$, sketch $f'$

(a)

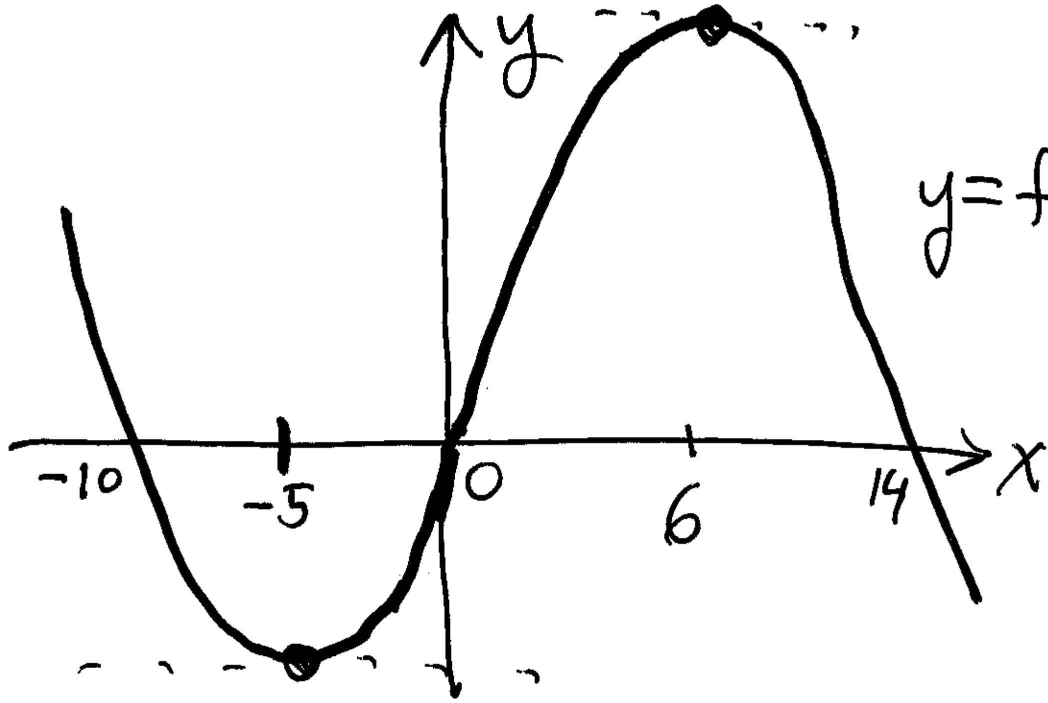


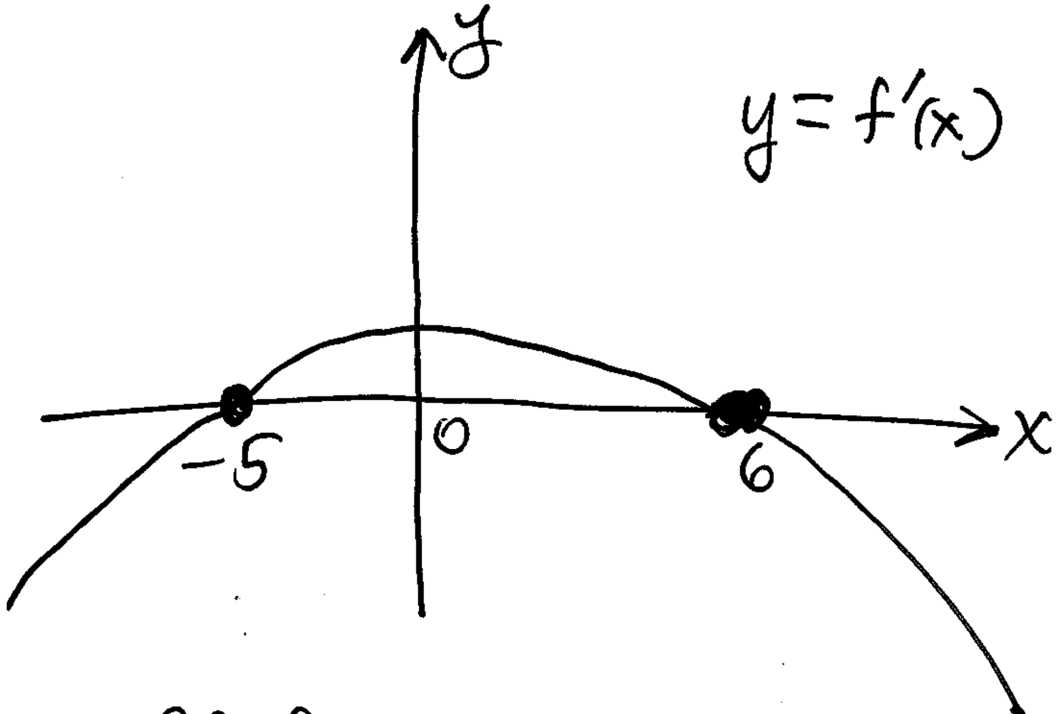




$f'$ DNE at $x = -1, 0, 3$

(b)



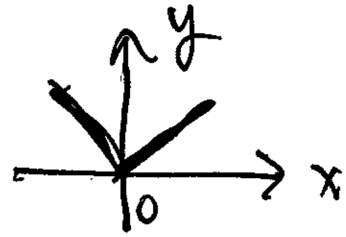



Theorem: If f is differentiable at a
$\Rightarrow$ f is cont. at a
i.e. If f is not cont at a $\Rightarrow$ f not diff at a)

**f is not diff at a :**

- if f is not cont at a
- if f has corners




- if f has vertical tangent at a