How to be Unsuccessful

$\boxed{1}$ I stopped attending class.

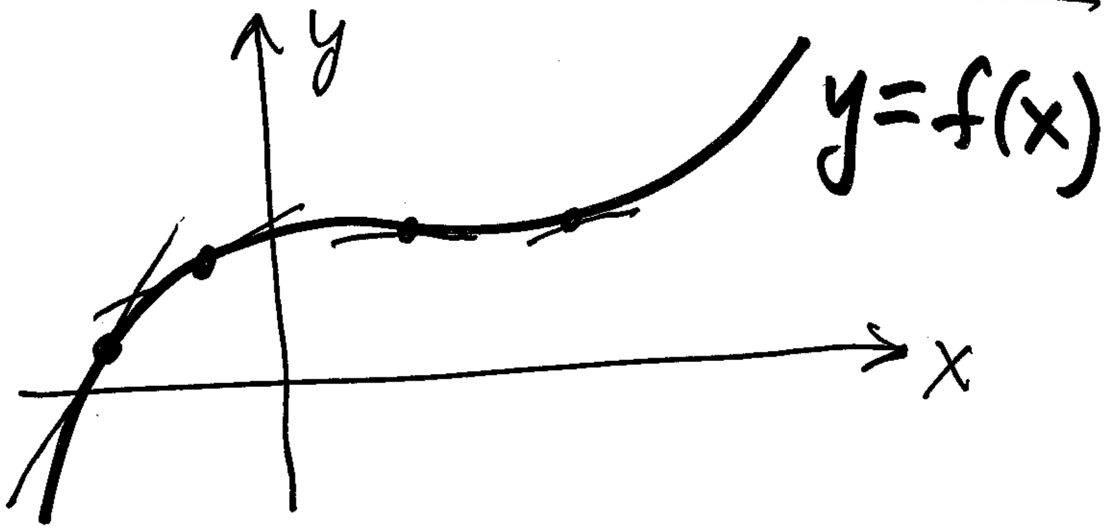
$\boxed{2}$ I neither read the book nor took notes.

$\boxed{3}$ I never learned the material.

$\boxed{4}$ I am a JIT student.

$\boxed{5}$ I thought I was smart.

§3.2 – The derivative as a function



$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \quad \left(\text{or } f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}\right.$$

Hence

<u>Def</u>: The <u>derivative</u> of f is the function

$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

provided limit exists and x in domain of f.

We say f is <u>differentiable</u> at x

(else f is <u>not</u> differentiable at x)

$\boxed{\text{Ex1}}$ Compute $f'(x)$ if $f(x) = 4x + \frac{1}{\sqrt{x}}$.

Soln: $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$$= \lim_{h \to 0} \frac{\left[4(x+h) + \frac{1}{\sqrt{x+h}}\right] - \left[4x + \frac{1}{\sqrt{x}}\right]}{h}$$

$$= \lim_{h \to 0} \frac{\left[4h + \left(\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}\right)\right]}{h}$$

$$= \underbrace{\lim_{h \to 0} \frac{4\cancel{h}}{\cancel{h}}} + \underbrace{\lim_{h \to 0} \frac{\left(\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}\right)}{h}}$$

provided these exist!

$$\therefore \lim_{h \to 0} \frac{\frac{\sqrt{x} - \sqrt{x+h}}{(\sqrt{x+h})\sqrt{x}}}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{x} - \sqrt{x+h}}{h(\sqrt{x+h})\sqrt{x}} \left(\frac{\sqrt{x} + \sqrt{x+h}}{\sqrt{x} + \sqrt{x+h}}\right)$$

$$= \lim_{h \to 0} \frac{-\cancel{h}}{\cancel{h}\left(\sqrt{x+h}\right)\sqrt{x}\left(\sqrt{x}+\sqrt{x+h}\right)}$$

$$= \frac{-1}{x\left(2\sqrt{x}\right)}$$

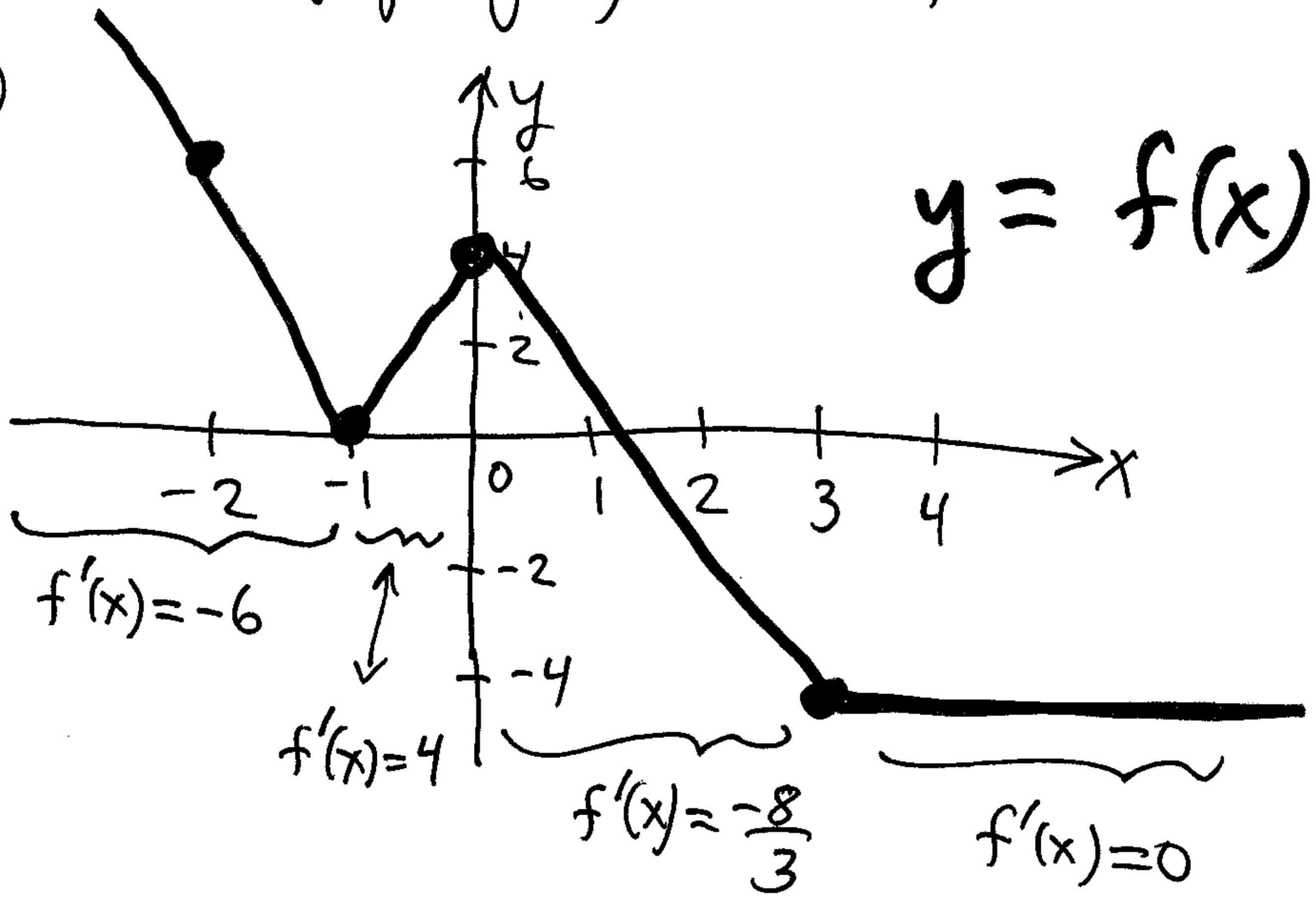
$$\therefore f'(x) = 4 - \frac{1}{2x^{3/2}}$$

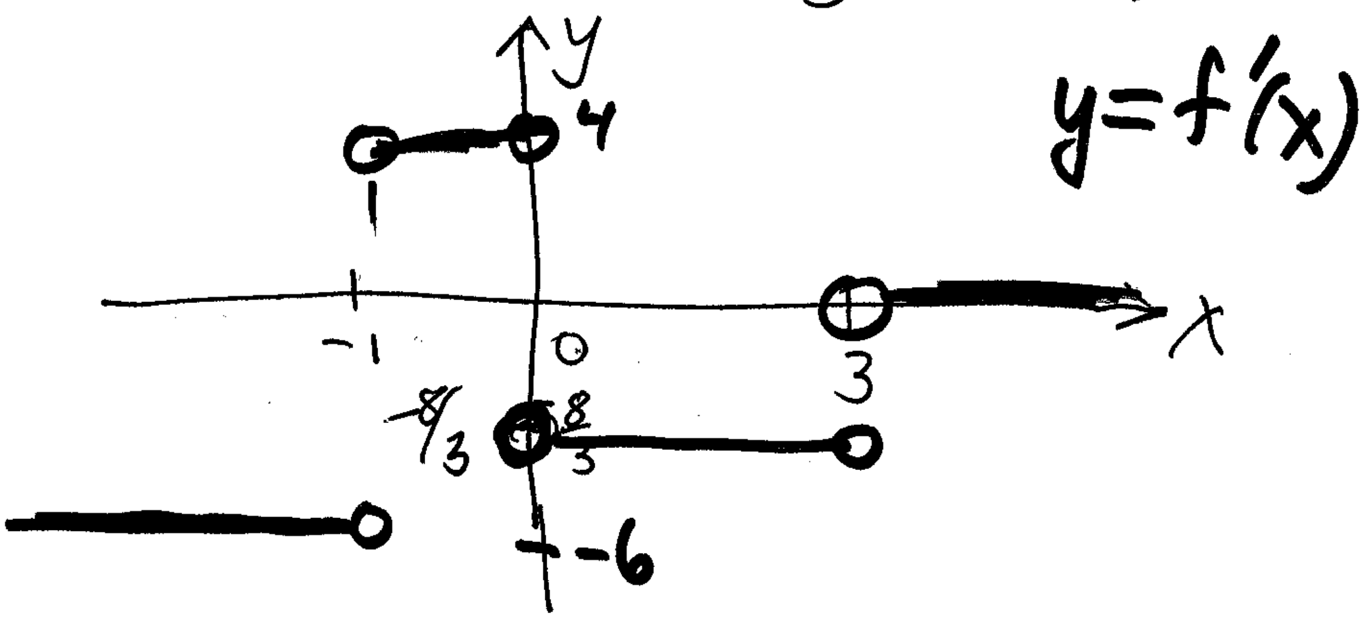
<u>Notation</u>: $y = f(x)$, then the derivative is:

$f'(x)$, $\frac{df}{dx}$, y', $\frac{dy}{dx}$

$\boxed{\text{Ex 2}}$ Given graph of f, sketch f'

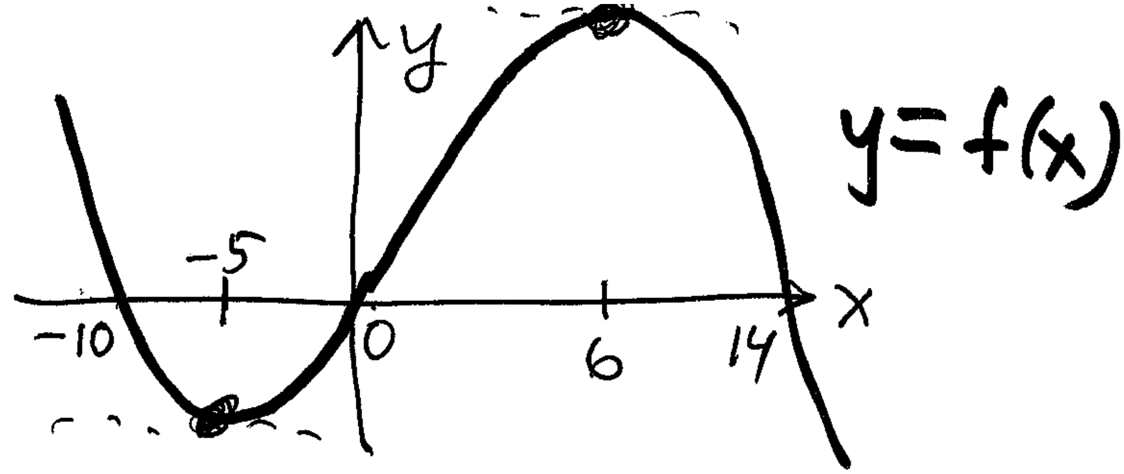
(a)

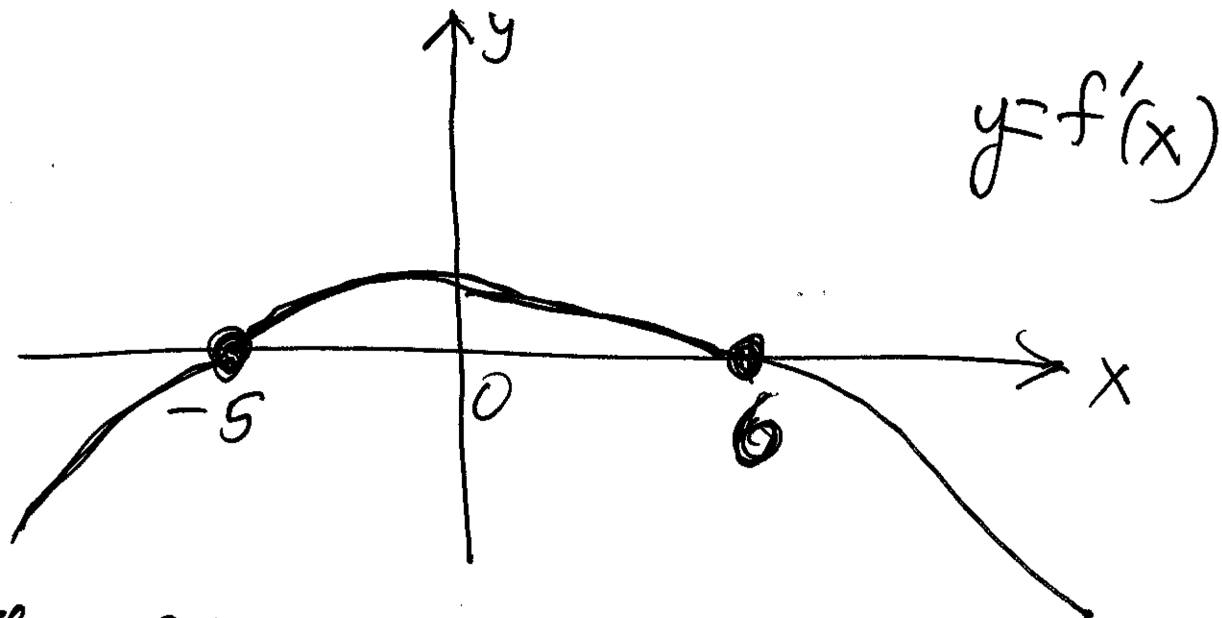




f' DNE at $x = -1, 0, 3$

(b)



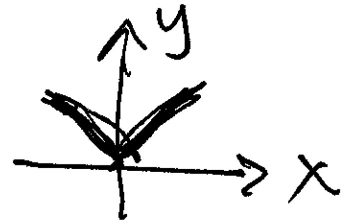


<u>Thm</u>: If f is diff. at $a \Rightarrow f$ cont at a

(i.e., If f not cont. at $a \Rightarrow f$ not diff at a)

f is <u>not</u> diff at a if

- f not cont. at a
- f has corners



- f has vertical tangent