Review #1

BASIC PROBLEMS

#16, 20, 21, 25, 30, 31

EXTRA PROBLEM

#E3

$\boxed{\#16}$ $\boxed{y(t) = y_0 e^{kt}}$ 18 gms decays to 2 gms in 2 days (B16)

$\Rightarrow y_0 = 18$ so $y(t) = 18e^{kt}$

Now $2 = y(2) = 18e^{k(2)} \Rightarrow \frac{1}{9} = e^{2k}$

$\Rightarrow k = \frac{1}{2}\ln\left(\frac{1}{9}\right)$ ✓

(Note: Half-life is $T_{\frac{1}{2}} = \dfrac{\ln\left(\frac{1}{2}\right)}{k} = \dfrac{\ln\left(\frac{1}{2}\right)}{\left\{\frac{1}{2}\ln\left(\frac{1}{9}\right)\right\}} = \dfrac{2\ln 2}{\ln 9}$)

Now $\underline{y(t) = y_0 e^{\frac{t}{2}\ln\left(\frac{1}{9}\right)}}$

If $y(0) = 12 = y_0$ so $y(t) = 12e^{\frac{t}{2}\ln\left(\frac{1}{9}\right)}$ ✓

Hence find t so that $y(t) = 4$

$\therefore 4 = y(t) = 12e^{\frac{t}{2}\ln\left(\frac{1}{9}\right)} \Rightarrow \frac{1}{3} = e^{\frac{t}{2}\ln\left(\frac{1}{9}\right)}$

$\Rightarrow t = \dfrac{\ln\left(\frac{1}{3}\right)}{\left\{\frac{1}{2}\ln\left(\frac{1}{9}\right)\right\}} = \dfrac{2\ln\left(\frac{1}{3}\right)}{\ln\left(\frac{1}{9}\right)} = \dfrac{\ln\left(\frac{1}{9}\right)}{\ln\left(\frac{1}{9}\right)} = 1$

$\boxed{B}$

$\boxed{\#20}$

B20

$$\frac{d}{dx}\left\{\int_1^{2x} \sqrt{t^2+1}\, dt\right\}$$

Use Leibnitz' Rule i.e. FTC, Part I

$$= \left(\sqrt{(2x)^2+1}\right)(2)$$

$$= \left(\sqrt{4x^2+1}\right)(2)$$

Hence when $x = \sqrt{2}$

$$\Rightarrow \left(\sqrt{4(\sqrt{2})^2+1}\right)(2) = \left(\sqrt{9}\right)(2) = 6$$

$\boxed{A}$

$\boxed{\#21}$ (821)

$$\int_3^4 x\sqrt{25-x^2}\,dx = \int_3^4 x\sqrt{25-x^2}\,dx$$

let $\boxed{u = 25 - x^2}$

then $\frac{du}{dx} = -2x$

$\therefore \boxed{du = -2x\,dx}$

$$= \int_{16}^{9} \sqrt{u}\left(\frac{du}{-2}\right)$$

$$= -\frac{1}{2}\int_{16}^{9} u^{1/2}\,du$$

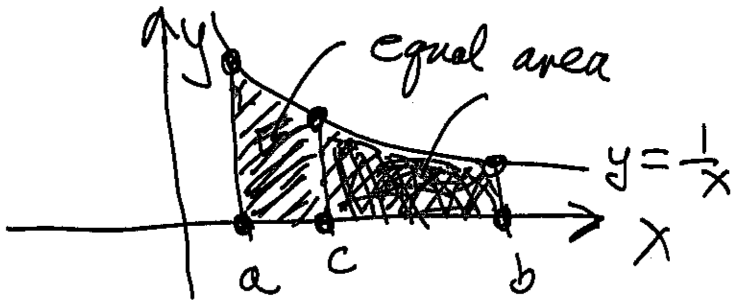
$$= \left(-\frac{1}{2}\,\frac{u^{3/2}}{3/2}\right)\Bigg|_{u=16}^{9} = \left(-\frac{1}{3}\,u^{3/2}\right)\Bigg|_{u=16}^{9}$$

$$= \left(-\frac{1}{3}\,9^{3/2}\right) - \left(-\frac{1}{3}\,16^{3/2}\right)$$

$$= \left(-\frac{1}{3}\cdot 27\right) - \left(-\frac{1}{3}\cdot 64\right) = \frac{37}{3}$$

$\boxed{C}$

#25



B25

$$\therefore \int_a^c \frac{1}{x}\,dx = \int_c^b \frac{1}{x}\,dx$$

$$\ln|x| \Big|_{x=a}^{c} = \ln|x| \Big|_{x=c}^{b}$$

$$\ln c - \ln a = \ln b - \ln c$$

$$2\ln c = \ln a + \ln b$$

$$2\ln c = \ln(ab)$$

$$\ln c = \frac{1}{2}\ln(ab) = \ln\left\{(ab)^{1/2}\right\}$$

$$\therefore\ c = \sqrt{ab}$$

$\boxed{A}$

$\boxed{\#30}$ B30

$$\int_0^{\sqrt{3}} \frac{1}{\sqrt{4-x^2}}\,dx = \int_0^{\sqrt{3}} \frac{1}{\sqrt{4\left(1-\frac{x^2}{4}\right)}}\,dx$$

$$= \frac{1}{2}\int_0^{\sqrt{3}} \frac{1}{\sqrt{1-\left(\frac{x}{2}\right)^2}}\,dx = \frac{1}{2}\int_0^{\sqrt{3}/2} \frac{1}{\sqrt{1-u^2}}\,(2du)$$

let $\boxed{u=\frac{x}{2}}$

$\frac{du}{dx}=\frac{1}{2}$

$\therefore \boxed{du=\frac{1}{2}dx}$

$$= \frac{2}{2}\int_0^{\sqrt{3}/2} \frac{1}{\sqrt{1-u^2}}\,du$$

$$= \left(\sin^{-1} u\right)\Big|_{u=0}^{\sqrt{3}/2}$$

$$= \left(\sin^{-1}\frac{\sqrt{3}}{2}\right) - \left(\sin^{-1} 0\right)$$

$$= \left(\frac{\pi}{3}\right) - 0 = \frac{\pi}{3}$$

$\boxed{D}$

$\boxed{\#31}$ (B31)

$$\int_0^4 \frac{x}{\sqrt{1+2x}}\,dx = \int_1^9 \frac{\frac{u-1}{2}}{\sqrt{u}}\left(\frac{du}{2}\right)$$

let $\boxed{u = 1+2x}$

$\frac{du}{dx} = 2$

$\therefore \boxed{du = 2\,dx}$

since $u = 1+2x$

$\Rightarrow \frac{u-1}{2} = x$

$$= \frac{1}{4}\int_1^9 \frac{u-1}{u^{1/2}}\,du$$

$$= \frac{1}{4}\int_1^9 \left(u^{1/2} - u^{-1/2}\right) du$$

$$= \frac{1}{4}\left(\frac{u^{3/2}}{3/2} - \frac{u^{1/2}}{1/2}\right)\Bigg|_{u=1}^{9}$$

$$= \frac{1}{4}\left(\frac{2u^{3/2}}{3} - 2u^{1/2}\right)\Bigg|_{u=1}^{9}$$

$$= \left\{\frac{1}{4}\left(\frac{2\cdot 9^{3/2}}{3} - 2\cdot 9^{1/2}\right)\right\} - \left\{\frac{1}{4}\left(\frac{2\cdot 1^{3/2}}{3} - 2\cdot 1^{1/2}\right)\right\}$$

$$= \left\{\frac{1}{4}\left(\frac{54}{3} - 6\right)\right\} - \left\{\frac{1}{4}\left(\frac{2}{3} - 2\right)\right\} = \frac{10}{3}$$

$\boxed{B}$

E3

Find all antiderivatives of $f(x) = (2x-1)^3 + 4x^2 \sin(2x^3)$

Answer: $\int f(x)dx = \underbrace{\int (2x-1)^3 dx}_{I} + \underbrace{\int 4x^2 \sin(2x^3)\,dx}_{J}$

$I = \int (2x-1)^3 dx = \int u^3 \left(\frac{du}{2}\right) = \frac{1}{2}\frac{u^4}{4} + C = \frac{1}{8}u^4 + C_1$

$= \frac{1}{8}(2x-1)^4 + C_1$ check!

$\boxed{u = 2x-1}$

$\frac{du}{dx} = 2$

$\therefore \boxed{du = 2dx}$

$J = \int 4x^2 \sin(2x^3)dx = \int 4x^2 \sin(2x^3)dx$

let $\boxed{u = 2x^3}$

$\frac{du}{dx} = 6x^2$

$\therefore \boxed{du = 6x^2 dx}$

$= \int 4 \sin u \left(\frac{du}{6}\right)$

$= -\frac{2}{3}\cos u + C_2$

$= -\frac{2}{3}\cos(2x^3) + C_2$ check!

$\therefore I + J = \boxed{\frac{1}{8}(2x-1)^4 - \frac{2}{3}\cos(2x^3) + C}$