Review #2

See attached pages

$\boxed{\#3}$ $\int_0^2 x^2 e^{x^3}\,dx = \int_0^2 x^2 e^{x^3}\,dx$

let $\boxed{u = x^3}$

then $\frac{du}{dx} = 3x^2$

so $\boxed{du = 3x^2\,dx}$

$= \int_0^8 e^u \left(\frac{du}{3}\right)$

$= \frac{1}{3} e^u \Big|_{u=0}^{8}$

$= \frac{1}{3}(e^8) - \frac{1}{3}(e^0)$

$= \frac{1}{3}(e^8 - 1)$ $\boxed{A}$

$\boxed{\#5}$ $$y(x) = \int_3^{\tan x} \sqrt{\sqrt{t} + 6t}\, dt$$

FTC (Part I) i.e. Leibnitz' Rule and Chain Rule give:

$$y'(x) = \left(\sqrt{\sqrt{\tan x} + 6\tan x}\right)\left(\sec^2 x\right)$$

$$\therefore\ y'\left(\tfrac{\pi}{4}\right) = \left(\sqrt{\sqrt{\tan\tfrac{\pi}{4}} + 6\tan\tfrac{\pi}{4}}\right)\left(\sec^2 \tfrac{\pi}{4}\right)$$

$$= \left(\sqrt{1+6(1)}\right)\left(\frac{2}{\sqrt{2}}\right)^2 \qquad \cos\tfrac{\pi}{4} = \tfrac{\sqrt{2}}{2}$$

$$= \left(\sqrt{7}\right)\left(\frac{4}{2}\right) = 2\sqrt{7} \quad \boxed{B}$$

$\boxed{\#8}$ $\boxed{y(t) = y_0 e^{kt}}$ 40% decays in 50 days.

i.e. 60% remains after 50 days; i.e. $0.60 y_0$

Hence $0.60 \cancel{y_0} = y(50) = \cancel{y_0} e^{k(50)}$

$\Rightarrow \ln(0.60) = k(50)$

$\Rightarrow k = \dfrac{\ln(0.60)}{50}$

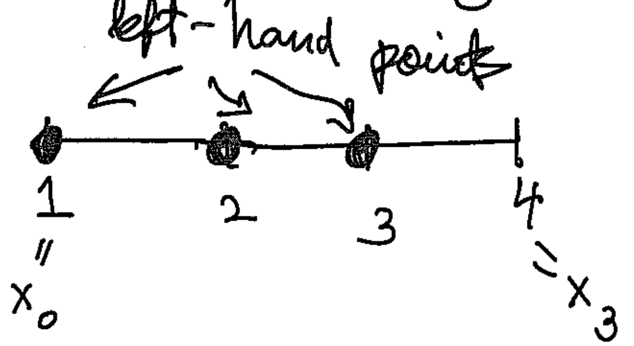
$\therefore$ Half-life is $T_{\frac{1}{2}} = \dfrac{\ln\left(\frac{1}{2}\right)}{k} = \dfrac{\ln\left(\frac{1}{2}\right)}{\left\{\frac{\ln(0.60)}{50}\right\}}$

So $T_{\frac{1}{2}} = 50 \dfrac{\ln(0.50)}{\ln(0.60)}$ $\boxed{B}$

$\boxed{\#9}$ $y = f(x) = \sqrt{x}$, $[\underset{a}{1}, \underset{b}{4}]$

$n = 3$

$\therefore \Delta x = \frac{b-a}{n} = \frac{4-1}{3} = 1$ ✓



Left Riemann Sum

$\therefore \sum_{k=1}^{3} f(x_k^*)\Delta x = \{f(1)\cdot 1\} + \{f(2)\cdot 1\} + \{f(3)\cdot 1\}$

$= \underline{\sqrt{1} + \sqrt{2} + \sqrt{3}}$ $\boxed{A}$

Find all antiderivatives of $\boxed{A1}$

$$f(x) = \frac{3}{x} + \frac{1}{x^{2/3}} - 1$$

<u>Soln</u>: Find $F(x)$ so that $F'(x) = f(x)$, hence

$$F(x) = \int \left(\frac{3}{x} + \frac{1}{x^{2/3}} - 1\right) dx$$

$$= \int \frac{3}{x}\,dx + \int x^{-2/3}\,dx - \int 1\,dx$$

$$= 3\ln|x| + \frac{x^{-\frac{2}{3}+1}}{-\frac{2}{3}+1} - x + C$$

$$\therefore F(x) = 3\ln|x| + 3x^{1/3} - x + C$$

$\boxed{\#23}$

B23

$$\frac{0}{0}$$

$$\lim_{x\to 0} \frac{2x-\sin^{-1}x}{2x+\tan^{-1}x} \underset{\substack{\uparrow \\ LR}}{=} \lim_{x\to 0} \frac{2-\frac{1}{\sqrt{1-x^2}}}{2+\frac{1}{1+x^2}}$$

$$= \frac{2-1}{2+1} = \frac{1}{3} \quad \boxed{C}$$

$\boxed{\#4}$ $\lim_{x \to 0^+} x \cos \frac{1}{x}$ (B4)

Must use Squeeze/Sandwich Theorem:

$$-x \leq x \cos\left(\frac{1}{x}\right) \leq x \qquad \text{for } x \neq 0$$

$\lim_{x\to 0^+}$ 0 $\lim_{x\to 0^+}$ 0

$$\therefore \lim_{x \to 0^+} x \cos\left(\frac{1}{x}\right) = 0$$

$\boxed{A}$

$$\lim_{x\to\infty} \frac{\sqrt{4x^2+1}}{x} \quad \text{and} \quad \lim_{x\to-\infty} \frac{\sqrt{4x^2+1}}{x}$$

E1

let $t = \frac{1}{x}$ $\Rightarrow$ $\frac{\sqrt{4x^2+1}}{x} = \frac{\sqrt{4\left(\frac{1}{t}\right)^2+1}}{\frac{1}{t}}$

$$= t\sqrt{\frac{4}{t^2}+1} = t\sqrt{\frac{4+t^2}{t^2}} = \frac{t\sqrt{4+t^2}}{\sqrt{t^2}} = \frac{t\sqrt{4+t^2}}{|t|}$$

$$\therefore \lim_{x\to\infty} \frac{\sqrt{4x^2+1}}{x} = \lim_{t\to 0^+} \frac{t\sqrt{4+t^2}}{t} = 2 \checkmark$$

$$\lim_{x\to-\infty} \frac{\sqrt{4x^2+1}}{x} = \lim_{t\to 0^-} \frac{t\sqrt{4+t^2}}{(-t)} = -2 \checkmark$$

Recall,

$$\sqrt{t^2} = |t| = \begin{cases} t, & \text{if } t \geq 0 \\ -t, & \text{if } t < 0 \end{cases}$$

$\boxed{\#4}$ Spring 2019 : $f(x) = \dfrac{e^x+1}{e^x-1}$ (E4)

<u>Vertical asymptote</u> where $e^x - 1 = 0$:

Since $\lim\limits_{x\to 0^+}$, $\lim\limits_{x\to 0^-}$, $\lim\limits_{x\to 0}$ At least 1 DNE

$e^x = 1 \Rightarrow x = \ln 1 = 0$ ✓

$\therefore \boxed{x=0}$

<u>Horizontal asymptote</u> :

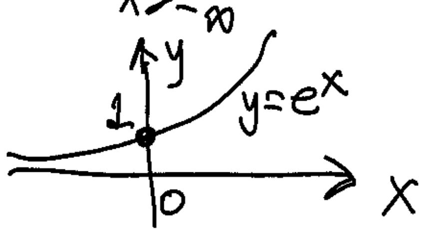
$$\lim_{x\to\infty} \frac{e^x+1}{e^x-1} \underset{\substack{\uparrow \\ LR}}{=} \lim_{x\to\infty} \frac{e^x}{e^x} = 1$$

$\therefore \boxed{y=1}$

Must also check

$$\lim_{x\to-\infty} \frac{e^x+1}{e^x-1} = \frac{1}{-1} = -1 \quad \therefore \boxed{y=-1}$$

Recall $\lim\limits_{x\to-\infty} e^x = 0$



$\boxed{D}$

$\boxed{\#10}$ $y = y(x)$ defined implicitly by

B10

$$xy^2 - x^2 + y + 5 = 0$$

$$\Rightarrow \frac{d}{dx}\left\{xy^2 - x^2 + y + 5\right\} = \frac{d}{dx}\{0\}$$

$$x\left\{2y\left(\frac{dy}{dx}\right)\right\} + y^2(1) - 2x + \left(\frac{dy}{dx}\right) + 0 = 0$$

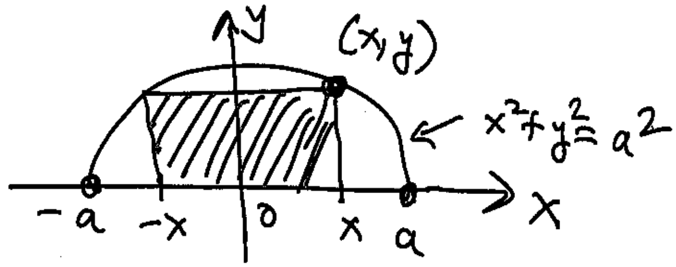
$$\frac{dy}{dx}\left(2xy + 1\right) = 2x - y^2$$

$$\Rightarrow \frac{dy}{dx} = \frac{2x - y^2}{2xy + 1}$$

$$\therefore \left.\frac{dy}{dx}\right|_{(x,y)=(-2,1)} = \frac{2(-2) - 1}{2(-2)(1) + 1} = \frac{5}{3}$$

$\boxed{E}$

$\boxed{\#14}$



B14

Maximize: $A = 2xy$

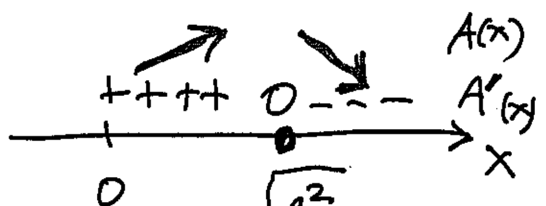
Constraint: $x^2+y^2=a^2$ Solve for y to get

$y = \sqrt{a^2-x^2}$ $\therefore$ Maximize $\underline{A(x) = 2x\sqrt{a^2-x^2}}$

Now $A'(x) = 2x\left(\frac{-x}{\sqrt{a^2-x^2}}\right) + 2\sqrt{a^2-x^2}$

$= \frac{-2x^2}{\sqrt{a^2-x^2}} + 2\sqrt{a^2-x^2}$

$= \frac{2}{\sqrt{a^2-x^2}}\left(-x^2+(a^2-x^2)\right) = \frac{2}{\sqrt{a^2-x^2}}\left(a^2-2x^2\right)$



← max area occurs when $x = \frac{\sqrt{a}}{2}$

$\therefore$ Max area is $A\left(\frac{a}{\sqrt{2}}\right) = 2\left(\frac{a}{\sqrt{2}}\right)\sqrt{a^2-\frac{a^2}{2}} = a^2$ $\boxed{E}$