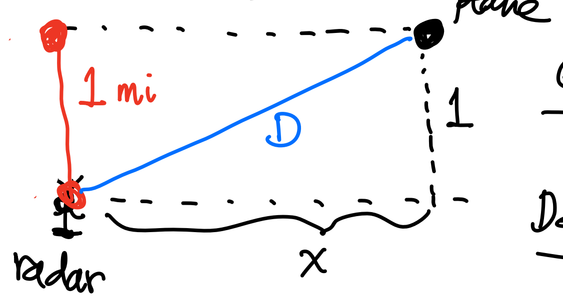
$\boxed{A}$

A plane is flying horizontally at an altitude of 1 mile at 500 mph. It passes directly over a radar station. What is rate at which the distance from plane to radar station is increasing when plane is 2 miles from station?



Given rate: $\frac{dx}{dt} = 500$

Desired rate: $\frac{dD}{dt}$ when $D = 2$

Equation: $D^2 = x^2 + 1^2$ ⊛

$$\Rightarrow \frac{d(D^2)}{dt} = \frac{d}{dt}(x^2) + \frac{d}{dt}(1)$$

$$2D\frac{dD}{dt} = 2x\frac{dx}{dt} \Rightarrow \frac{dD}{dt} = \frac{x}{D}\frac{dx}{dt}$$

Now when $D = 2$ ⊛ $\Rightarrow 2^2 = x^2 + 1 \Rightarrow x = \sqrt{3}$

$$\therefore \left.\frac{dD}{dt}\right|_{\substack{D=2 \\ x=\sqrt{3}}} = \frac{\sqrt{3}}{2}(500) = \underline{250\sqrt{3}} \text{ mph}$$

$\boxed{\text{SG1 \#17}}$ Vertical Asymptotes of $\boxed{B}$

$$f(x) = \frac{-x^2+16}{x^2+5x+4}$$

$$f(x) = \frac{16-x^2}{x^2+5x+4} = \frac{(4-x)(4+x)}{(x+4)(x+1)} = \underbrace{\frac{4-x}{x+1}}_{\text{Reduced Form of Rational function}}$$

$\therefore$ $\boxed{x=-1}$ Only vertical asymptote ✓

and since $\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} \frac{4-x}{x+1} \overset{\uparrow}{\underset{LR}{=}} -1$

and $\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to\infty} \frac{4-x}{x+1} \overset{\uparrow}{\underset{LR}{=}} -1$

$\therefore$ $\boxed{y=-1}$ Only horizontal asymptote ✓

Does $f(x) = \frac{x^3+x^2-1}{x-4}$ have horizontal asymptote?

$\boxed{\text{NO}}$ because $\lim\limits_{x\to\infty} f(x) = \infty$ (limits are not finite)

and $\lim\limits_{x\to-\infty} f(x) = \infty$

$\boxed{SG1 \# 23}$ $\boxed{C}$

$$\lim_{x \to 1}\left[\left(\frac{1-x}{1-\sqrt{x}}\right)^2\right] = \left[\lim_{x \to 1}\left(\frac{1-x}{1-\sqrt{x}}\right)\right]^2 \quad \frac{0}{0}$$

$$\underset{\uparrow \atop LR}{=} \left[\lim_{x \to 1}\frac{-1}{-\frac{1}{2\sqrt{x}}}\right]^2 = \left[\frac{-1}{-\frac{1}{2}}\right]^2 = [2]^2 = \underline{\underline{4}}$$

$\boxed{\text{SG2 \#9}}$ Find $f'(x)$ if $\boxed{D}$

$$f(x) = \frac{1}{\sqrt[3]{1-x^2}}$$

$$f(x) = \frac{1}{(1-x^2)^{1/3}} = (1-x^2)^{-1/3}$$

$$\therefore\ f'(x) = -\frac{1}{3}(1-x^2)^{-4/3}(-2x)$$

$$= \frac{2x}{3}(1-x^2)^{-4/3} \checkmark$$

$\boxed{E}$

Which of the following is/are true about $g(x) = 4x^3 - 3x^4$?

(1) g is decreasing for $x > 1$

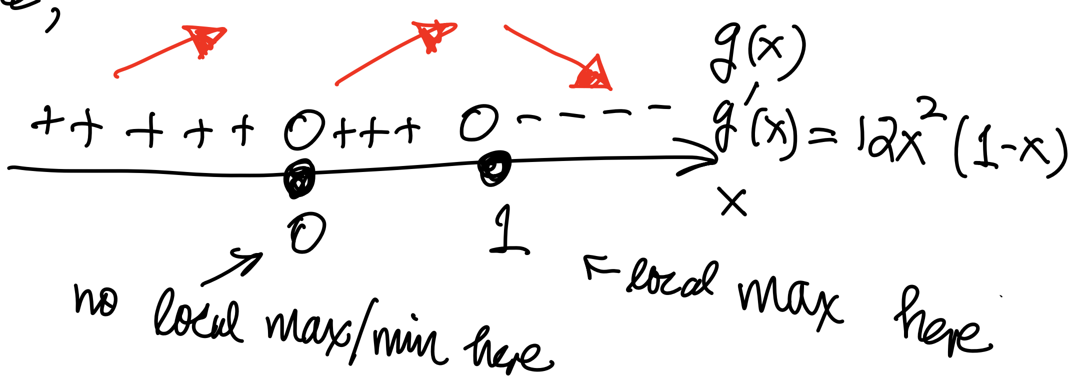
(2) g has relative extreme value at $(0,0)$

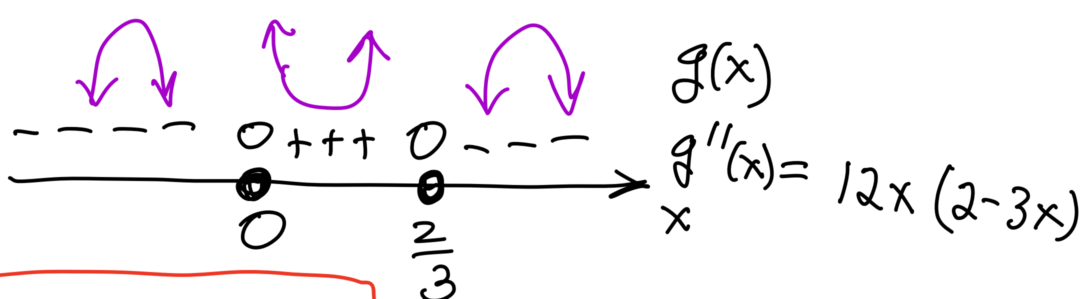
(3) g is concave up for all $x < 0$

$$g'(x) = 12x^2 - 12x^3 = 12x^2(1-x)$$

$$g''(x) = 24x - 36x^2 = 12x(2-3x)$$

Hence,





$\therefore$ (1) is TRUE
(2) is FALSE
(3) is FALSE

$$\lim_{x\to\infty} \frac{\sqrt{x^2+2x}+x}{x} \qquad \boxed{F}$$

Soln 1: $$\lim_{x\to\infty} \frac{\dfrac{\sqrt{x^2+2x}+x}{x}}{\dfrac{x}{x}} = \lim_{x\to\infty} \frac{\dfrac{\sqrt{x^2+2x}}{x}+\dfrac{x}{x}}{1}$$

$$= \lim_{x\to\infty}\left(\frac{\sqrt{x^2+2x}}{x}+1\right) = \lim_{x\to\infty}\left(\frac{\sqrt{x^2+2x}}{\sqrt{x^2}}+1\right)$$

$$= \lim_{x\to\infty}\left(\sqrt{\frac{x^2+2x}{x^2}}+1\right) = \lim_{x\to\infty}\left(\sqrt{\frac{x^2}{x^2}+\frac{2x}{x^2}}+1\right)$$

$$= \lim_{x\to\infty}\left(\sqrt{1+\frac{2}{x}}+1\right) = \underline{\underline{2}}$$

$$\lim_{x \to \infty} \frac{\sqrt{x^2+2x} + x}{x}$$

$\boxed{G}$

You can change of variables: let $t = \frac{1}{x}$

Then $x \to \infty$ if and only if $t \to 0^+$. Hence

$$\lim_{x \to \infty} \frac{\sqrt{x^2+2x} + x}{x} = \lim_{t \to 0^+} \frac{\sqrt{\left(\frac{1}{t}\right)^2 + \left(\frac{2}{t}\right)} + \left(\frac{1}{t}\right)}{\left(\frac{1}{t}\right)}$$

$$= \lim_{t \to 0^+} t\left\{\sqrt{\left(\frac{1}{t}\right)^2 + \left(\frac{2}{t}\right)} + \left(\frac{1}{t}\right)\right\}$$

$$= \lim_{t \to 0^+} t\left\{\sqrt{\frac{1}{t^2} + \frac{2}{t}} + \left(\frac{1}{t}\right)\right\}$$

$$= \lim_{t \to 0^+} t\left\{\sqrt{\frac{1+2t}{t^2}} + \left(\frac{1}{t}\right)\right\} = \lim_{t \to 0^+} t\left\{\frac{\sqrt{1+2t}}{|t|} + \left(\frac{1}{t}\right)\right\}$$

$$= \lim_{t \to 0^+} t\left\{\frac{\sqrt{1+2t}}{t} + \left(\frac{1}{t}\right)\right\} = \lim_{t \to 0^+} \left\{\sqrt{1+2t} + 1\right\} = \underline{\underline{2}}$$

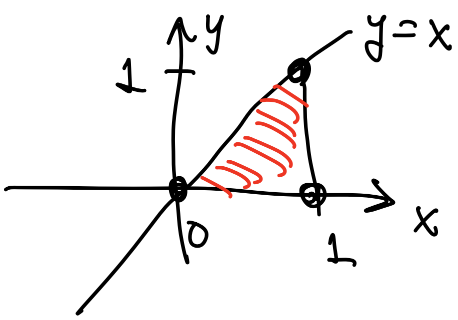
$\boxed{\#20}$ Fall 2017 Final Exam $\boxed{H_1}$

Find $\int_0^1 \left(x - \sqrt{1-x^2}\right) dx$

Hint: Interpret in terms of areas of basic shapes.

$$\int_0^1 \left(x - \sqrt{1-x^2}\right) dx = \underbrace{\int_0^1 x\, dx}_{I} - \underbrace{\int_0^1 \sqrt{1-x^2}\, dx}_{J}$$

$I = \int_0^1 x\,dx =$ area under $y = x$ over $[0, 1]$:



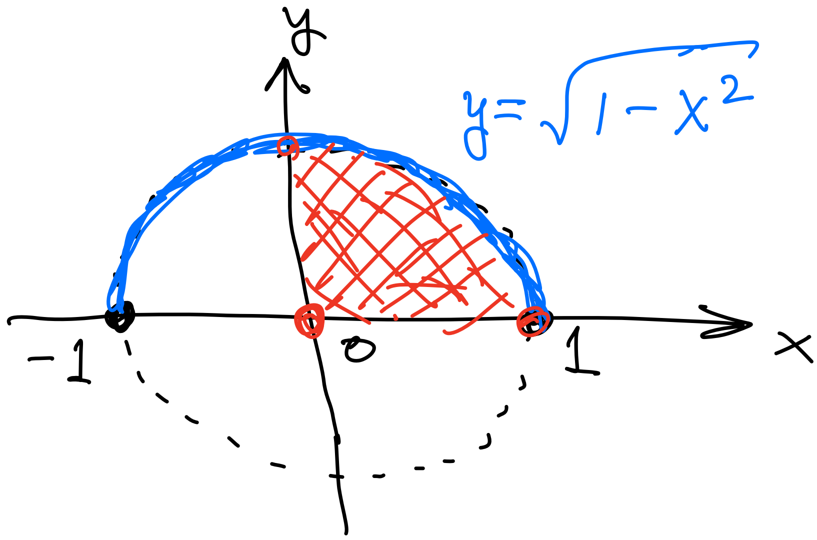
$\therefore I = \frac{1}{2}(1)(1) = \frac{1}{2}$

$J = \int_0^1 \sqrt{1-x^2}\, dx =$ area under curve $y = \sqrt{1-x^2}$ over $[0, 1]$

this is upper part of the circle $x^2 + y^2 = 1$

(cont'd)

$\boxed{H_2}$



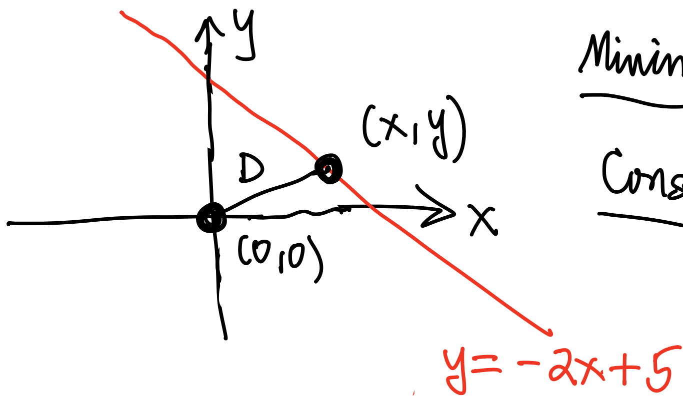
$\therefore \quad J = \frac{1}{4}$ (area of circle of radius 1)

so $\quad J = \frac{1}{4}\left(\pi(1)^2\right) = \frac{\pi}{4}$

$$\therefore \int_0^1 \left(x - \sqrt{1-x^2}\right) dx = \frac{1}{2} - \frac{\pi}{4}$$

$\boxed{\#17}$ Fall 2017 Final Exam $\boxed{\text{I}}$

Find the minimum distance between the line $y=-2x+5$ and $(0,0)$



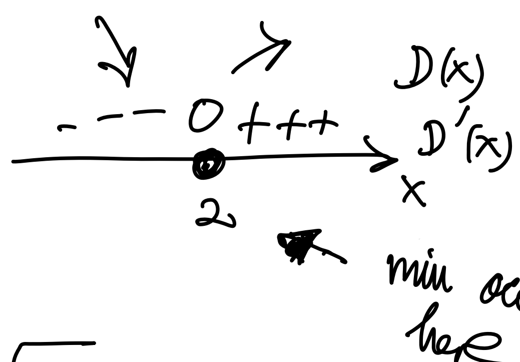
Minimize $D=\sqrt{(x-0)^2+(y-0)^2}$

Constraint: $\underbrace{y=-2x+5}$ use this to eliminate y from

$\therefore$ Minimize $D(x)=\sqrt{x^2+(-2x+5)^2}$

$$\Rightarrow \quad D'(x)=\frac{1}{2}\left(x^2+(-2x+5)^2\right)^{-1/2}\left\{2x+2(-2x+5)(-2)\right\}$$

$$D'(x)=\frac{5x-10}{\sqrt{x^2+(-2x+5)}} \Rightarrow$$



$\therefore$ Min distance is $D(2)=\sqrt{4+(1)^2}=\underline{\sqrt{5}}$

Find equation of tangent line to $x^3 e^{2y} + y^2 = x^2 + 4$ at $(2,0)$ $\boxed{J}$

$$x^3 e^{2y} + y^2 = x^2 + 4$$

$$\Rightarrow \frac{d}{dx}\left\{x^3 e^{2y} + y^2\right\} = \frac{d}{dx}\left\{x^2 + 4\right\}$$

$$x^3\left\{e^{2y}\left(2\frac{dy}{dx}\right)\right\} + e^{2y}\left\{3x^2\right\} + 2y\frac{dy}{dx} = 2x$$

$$\frac{dy}{dx}\left\{2x^3 e^{2y} + 2y\right\} = 2x - 3x^2 e^{2y}$$

$$\left.\frac{dy}{dx}\right|_{(x,y)=(2,0)} = \left.\frac{2x - 3x^2 e^{2y}}{2x^3 e^{2y} + 2y}\right|_{(x,y)=(2,0)}$$

$$= \frac{4-12}{16} = -\frac{1}{2}$$

$\therefore$ Tangent line is $y - 0 = -\frac{1}{2}(x-2)$

$$y = -\frac{x}{2} + 1$$

$\boxed{B\#6}$ The equation $x^3-x-5=0$ $\boxed{K}$

has one root between $x=-2$ and $x=2$.

The root is in the interval

A. $(-2,-1)$ B. $(-1,0)$ C. $(0,1)$

D. $(1,2)$ E. $(-1,1)$

Intermediate Value Thm: If f is cont. on $[a,b]$ and L is between $f(a)$ and $f(b)$, then there is a number c, $a<c<b$, where $f(c)=L$

For problem above, let $f(x)=x^3-x-5$, $L=0$ and now test each interval $[-2,-1]$, $[-1,0]$, $[0,1]$, $[1,2]$ and $[-1,1]$:

For eg $[\underset{a}{-2},\underset{b}{-1}] \Rightarrow f(-2)=-11$
$f(-1)=-5$ $\therefore$ No root in $(-2,-1)$

etc.... But, considering the interval $[\underset{a}{1},\underset{b}{2}]$

$\Rightarrow f(1)=-5$
$f(2)=5$ and since $L=0$ is between $f(1), f(2)$, there must be a zero in $(1,2)$

Which function has a removable discontinuity at $x=-3$? $\boxed{L}$

A. $f(x)=\frac{x^2-9}{x-3}$ B. $f(x)=\frac{1}{\sqrt{x+3}}$

C. $f(x)=\frac{x^2-9}{x+3}$ D. $f(x)=\ln(x+3)$

E. $f(x)=\sqrt[3]{x+3}$

$x=a$ is a removable discontinuity of $f(x)$ if

→ f is not cont at $x=a$, but $\lim_{x\to a} f(x)$ exists and is finite

Thus, check each function:

A. $f(x)=\frac{x^2-9}{x-3}$ is cont at $x=-3$ $\boxed{NO}$

B. $\lim_{x\to -3} f(x) = \lim_{x\to -3} \frac{1}{\sqrt{x+3}}$ DNE $\boxed{NO}$

C. $\lim_{x\to -3} f(x) = \lim_{x\to -3} \frac{x^2-9}{x+3} = -6$ and $f(x)$ not cont. at $x=3$

$\boxed{YES}$

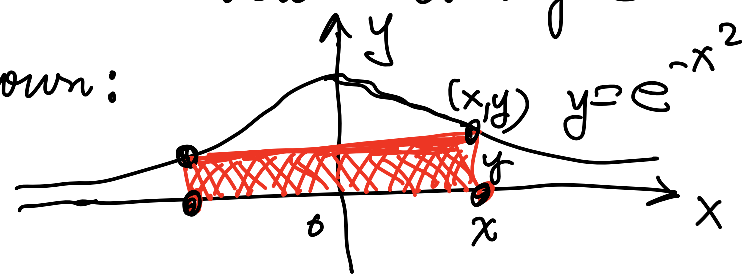
D. $\lim_{x\to -3} f(x)$ DNE $\boxed{NO}$

E. $f(x)=(x+3)^{1/3}$ is cont. at $x=-3$ $\boxed{NO}$

#18 Spring 2019 Final Exam

M

Find area of largest inscribed rectangle under $y = e^{-x^2}$ as shown:



$\underline{\text{Maximize}}$: $A = 2xy$

$\underline{\text{Constraint}}$: $y = e^{-x^2}$

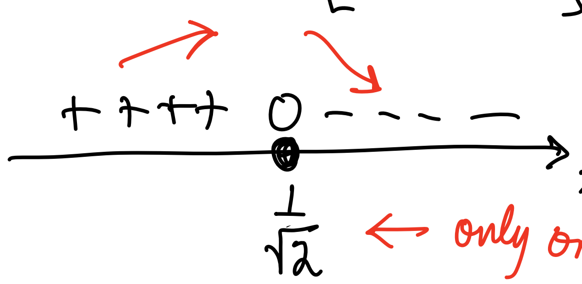
plug in to eliminate one variable

$\therefore \underbrace{A(x) = 2xe^{-x^2}}$

Maximize over $-\infty < x < \infty$

$$A'(x) = 2x\{e^{-x^2}(-2x)\} + e^{-x^2}\{2\}$$

$$A'(x) = 2e^{-x^2}[-2x^2 + 1]$$



← only one absolute extremum

$\therefore$ Largest area occurs when $x = \frac{1}{\sqrt{2}}$

so $A\left(\frac{1}{\sqrt{2}}\right) = \frac{2}{\sqrt{2}} e^{-\frac{1}{2}} = \sqrt{2}e^{-\frac{1}{2}} = \frac{\sqrt{2}}{e^{1/2}} = \sqrt{\frac{2}{e}}$ ✓

#8, Fall 2017

$\boxed{N}$

Find $f'(1)$ if $f(x) = \ln\left[\dfrac{(3x-1)^6}{(x+1)^4(2x+1)^3}\right]$

$$f(x) = \ln\{(3x-1)^6\} - \ln\{(x+1)^4(2x+1)^3\}$$

$$f(x) = 6\ln(3x-1) - [4\ln(x+1) + 3\ln(2x+1)]$$

$$f(x) = 6\ln(3x-1) - 4\ln(x+1) - 3\ln(2x+1)$$

$$\therefore f'(x) = 6\left(\frac{1}{3x-1}\right)(3) - 4\left(\frac{1}{x+1}\right)(1) - 3\left(\frac{1}{2x+1}\right)(2)$$

So $f'(1) = 6\left(\frac{1}{2}\right)(3) - 4\left(\frac{1}{2}\right)(1) - 3\left(\frac{1}{3}\right)(2)$

$$= 9 - 2 - 2 = \underline{\underline{5}}$$