5. If $f(x) = 3 + \sqrt{2 + 7x}$, find $f^{-1}(5)$

A. $\frac{1}{7}$ B. $\frac{2}{7}$ C. $\frac{3}{7}$ D. $\frac{4}{7}$ E. $\frac{5}{7}$

$$y = 3 + \sqrt{2+7x} \Rightarrow (y-3)^2 = \left(\sqrt{2+7x}\right)^2$$

$$(y-3)^2 = 2 + 7x \Rightarrow \frac{(y-3)^2 - 2}{7} = x$$

$$\therefore f^{-1}(x) = \frac{(x-3)^2 - 2}{7}$$

$$f^{-1}(5) = \frac{2}{7}$$

6. Which of the following does **NOT** have an inverse function?

A. $y = \sin x,\ -\frac{\pi}{2} \le x \le \frac{\pi}{2}$ B. $y = x^3 + 2$ C. $y = \dfrac{x}{x^2+1}$

D. $y = \frac{1}{2}e^x$ E. $y = \ln(x-2),\ x > 2$

7. The graph of a function g is obtained from the graph if f by first compressing vertically by a factor of $\mathbf{3}$, then shifting to the right by 2units, and then shifting up by $\mathbf{1}$ unit. What is $g(x) =$?

A. $f\left(\frac{x}{3}+1\right)+2$ B. $f\left(\frac{x+3}{3}+1\right)$ C. $\frac{1}{3}f(x-2)+1$

D. $3f(x+2)-1$ E. $f(3(x-2))-1$

8. Solve $\ln(x^2-9)-\ln(x-3)=2$

A. e^2-3 B. e^2+3 C. $\dfrac{1}{e^2}-3$ D. $\dfrac{1}{e^2}+3$ E. $e+3$

$$\ln\left(\frac{x^2-9}{x-3}\right)=2$$

$$\Rightarrow e^{\ln\left(\frac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}}\right)}=e^{2}$$

$$e^{\ln(x+3)}=e^2$$

$$x+3=e^2 \checkmark$$

9. Find all values of x in the interval $[0, 2\pi]$ satisfying

$2\sin x\cos x + \cos x = 0$

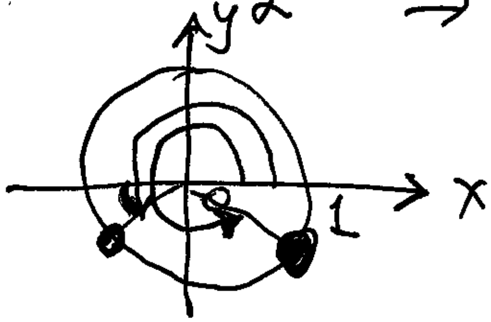
A. $\frac{\pi}{2}, \frac{3\pi}{2}$ B. $\frac{7\pi}{6}, \frac{11\pi}{6}$ C. $\frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$ D. $\frac{2\pi}{3}, \frac{4\pi}{3}$

E. No such values in given interval

$2\sin x\cos x + \cos x = 0 \Rightarrow (\cos x)(2\sin x + 1) = 0$

$\checkmark \cos x = 0 \Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$

$\checkmark \sin x = -\frac{1}{2}$



$\Rightarrow x = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$

$x = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$

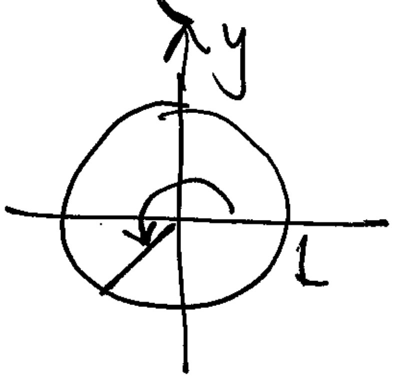
10. The altitude, in feet, of a drone t seconds after it takes off is given by $h(t) = \frac{t^3}{6} + 4$. What is the drone's average velocity in feet per second over the time interval $[0, 3]$?

A. 1 B. $\frac{3}{2}$ C. $\frac{9}{2}$ D. 9 E. 27

11. Evaluate $\sin^{-1}\left(\sin \frac{4\pi}{3}\right)$

A. $\frac{2\pi}{3}$ B. $-\frac{\pi}{3}$ C. $\frac{4\pi}{3}$ D. $-\frac{2\pi}{3}$ E. $\frac{\pi}{3}$

$$\sin^{-1}\left(\sin \frac{4\pi}{3}\right) = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$$



12. Write the trigonometric expression $\sin\left(\tan^{-1} u\right)$ as an algebraic expression in u.

A. $\frac{1}{\sqrt{1+u^2}}$ B. $\frac{u}{\sqrt{1+u^2}}$ C. $\frac{1}{\sqrt{u^2-1}}$ D. $\frac{u}{\sqrt{1-u^2}}$

E. $\frac{u}{\sqrt{u^2-1}}$

13. Find the limit: $\lim\limits_{x\to 1} \dfrac{x-1}{\sqrt{2x+23}-5}$ $\left(\dfrac{\sqrt{2x+23}+5}{\sqrt{2x+23}+5}\right)$

A. $\dfrac{5}{2}$ B. 5 C. $\dfrac{1}{2}$ D. 10 E. $\dfrac{2}{5}$

$$= \lim_{x\to 1} \frac{\cancel{(x-1)}\left(\sqrt{2x+23}+5\right)}{\underbrace{(2x+23-25)}_{2\cancel{(x-1)}}} = \lim_{x\to 1} \frac{\sqrt{2x+23}+5}{2}$$

$$=5$$

14. Find the limit: $\lim\limits_{x\to 0} x^2 \cos\left(\dfrac{2}{x}\right)$

A. 1 B. 0 C. $-\infty$ D. ∞ E. DNE

Squeeze Thm

$$x^2(-1) \le x^2 \cos\left(\frac{2}{x}\right) \le x^2(1)$$

As $x \to 0$

$$0 \quad \therefore \quad 0 \quad 0$$

$$0$$

15. Find the limit: $\displaystyle \lim_{t \to -2^+} \frac{t-1}{\sqrt{(t+3)(t+2)}}$

A. ∞ B. -3 C. 3 D. **DNE** and is neither ∞ nor $-\infty$

E. $-\infty$

Recall $|x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$

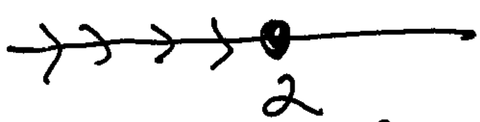
16. Find the limit: $\displaystyle \lim_{t \to 2^-} \frac{|2t-4|}{t^2-4}$

A. ∞ B. $-\frac{1}{2}$ C. $\frac{1}{2}$ D. **DNE** and is neither ∞ nor $-\infty$

E. $-\infty$

$$= \lim_{\substack{t \to 2^- \\ (t<2)}} \frac{|2t-4|}{t^2-4} = \lim_{t \to 2^-} \frac{-(2t-4)}{t^2-4} = \lim_{t \to 2^-} \frac{-2\cancel{(t-2)}}{\cancel{(t-2)}(t+2)}$$

$$= -\frac{1}{2}$$



$$|2t-4| = \begin{cases} (2t-4), & \text{if } (2t-4) \geq 0 \text{ i.e. } t \geq 2 \\ -(2t-4), & \text{if } (2t-4) < 0 \text{ i.e. } t < 2 \end{cases}$$

17. Find all vertical asymptotes of $f(x) = \dfrac{-x^2 + 16}{x^2 + 5x + 4}$

A. $x = -1$ B. $x = -1,\ x = 4$ C. $x = 1,\ x = -4$

D. $x = -1,\ x = -4$ E. $x = 4$

18. Suppose $f(x) = \dfrac{2x^3 + 16x^2 + 30x}{x^3 + 5x^2}$. Which of the following statements are **correct**?

(i) $y = 2$ is a horizontal asymptote Correct

(ii) $x = -5$ is a vertical asymptote NO

(iii) $\lim_{x \to 0} f(x) = \infty$ NO

A. Only statement (i) B. Only statements (i) and (ii) C. Only statements (i) and (iii) D. All three are correct E. Only statements (ii) and (iii)

$$f(x) = \frac{2x(x^2+8x+15)}{x^2(x+5)} = \frac{2x\cancel{(x+5)}(x+3)}{x^2\cancel{(x+5)}}$$

$$\boxed{f(x) = \frac{2x+6}{x}}$$

$$\lim_{x\to\infty} \frac{2x+6}{x} = 2$$

$$\lim_{x\to 0^+} \frac{2x+6}{x} = +\infty$$

$$\lim_{x\to 0^-} \frac{2x+6}{x} = -\infty$$

$\lim_{x\to 0} f(x)$ DNE and is not ∞ or $-\infty$

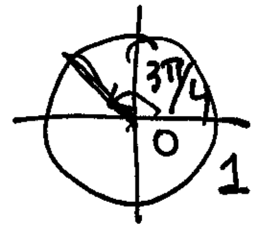
21. Find the value of c such that f is continuous at $x = 2$:

$$f(x) = \begin{cases} \dfrac{x^2 - 5x + c}{x - 2}, & \text{if } x < 2 \\[2ex] \tan\left(\dfrac{3\pi}{2x}\right), & \text{if } x \geq 2 \end{cases}$$

A. $c = 6$ B. No value of c C. $c = 3$ D. $c = -1$

E. $c = 4$

$$\underbrace{\lim_{x \to 2} f(x)}_{} = f(2)$$



$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = \tan\left(\frac{3\pi}{4}\right) = -1$$

$x < 2$ $x > 2$

2 2

$$\lim_{x \to 2^-} \frac{x^2 - 5x + c}{x - 2} = \lim_{x \to 2^+} \tan\left(\frac{3\pi}{2x}\right) = -1$$

$\Rightarrow x^2 - 5x + C = 0$ at $x = 2$

$\therefore 4 - 10 + C = 0$

$C = 6$ ✓

26. The quantity $\lim\limits_{h \to 0} \dfrac{\sqrt{9+h}-3}{h}$ represents which of the following?

A. $f'(3)$ with $f(x) = \sqrt{x}$ B. $f'(6)$ with $f(x) = \sqrt{x}$

C. $f'(-6)$ with $f(x) = \sqrt{x+3}$ D. $f'(-3)$ with $f(x) = \sqrt{x+9}$

E. $f'(6)$ with $f(x) = \sqrt{x+3}$

27. If $f(x) = \dfrac{8e^{x} + 3e^{3x}}{2e^{2x} - e^{3x}}$, which of the following statements are TRUE?

(I) $\lim\limits_{x \to \infty} f(x) = -3$

(II) $\lim\limits_{x \to -\infty} f(x) = \infty$

(III) $f(x)$ has a vertical asymptote at $x = \ln 2$

A. Only (I) B Only (I) and (II) C. Only (I) and (III)

D. Only (II) and (III) E. All three are true

SEE NEXT PAGE

FOR COMPLETE

SOLUTION...

$\boxed{\#27}$

$$f(x) = \frac{8e^x + 3e^{3x}}{2e^{2x} - e^{3x}}$$

As $x \to \infty$ (i.e. x large) the dominant term is e^{3x} so divide numerator & denominator by it:

$$f(x) = \frac{(8e^x + 3e^{3x})/e^{3x}}{(2e^{2x} - e^{3x})/e^{3x}} = \frac{\frac{8}{e^{2x}} + 3}{\frac{2}{e^x} - 1}$$

$\therefore \lim_{x\to\infty} f(x) = -3 \quad \therefore \boxed{\text{I}}$ TRUE

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{8e^x + 3e^{3x}}{2e^{2x} - e^{3x}} = \lim_{x\to-\infty} \frac{8\overset{\nearrow 0}{e^x} + 3\overset{\nearrow 0}{e^{3x}}}{\underset{\swarrow 0}{e^{2x}}\underset{\searrow 2}{(2 - e^x)}}$$

$= \infty \quad \therefore \boxed{\text{II}}$ TRUE (thru positive #'s)

Since $f(x) = \dfrac{8e^x + 3e^{3x}}{2e^{2x} - e^{3x}} \Rightarrow$ vertical asymptote

when $2e^{2x} - e^{3x} = 0$ so $2e^{2x} = e^{3x}$

$$\Rightarrow \frac{2e^{2x}}{e^{2x}} = \frac{e^{3x}}{e^{2x}} \Rightarrow 2 = e^x$$

$\ln 2 = x \quad \therefore \boxed{\text{III}}$ TRUE

Note: numerator $\neq 0$ at $x = \ln 2$

$\therefore \; x = \ln 2$ is vertical asymp.