(1) If $f(x) = 2x^2 + 4$, which of the following is $f'(3)$?

A. $\lim_{h\to 0} \frac{2(3+h)^2 - 10}{h}$ B. $\lim_{x\to 3} \frac{2x^2 - 18}{x-3}$

C. $\lim_{h\to 0} \frac{2(3+h)^2 + 18}{h}$ D. $\lim_{x\to 3} \frac{2x^2 + 18x}{x-3}$ E. $\lim_{h\to 0} \frac{2h^2 - 18}{h-3}$

$$f'(3) = \lim_{h\to 0} \frac{f(3+h) - f(3)}{h}$$

$$= \lim_{h\to 0} \frac{[2(3+h)^2+4] - [22]}{h}$$

$$= \lim_{h\to 0} \frac{2(3+h)^2 - 18}{h}$$

Try other form:

$$f'(3) = \lim_{x\to 3} \frac{f(x) - f(3)}{x-3} = \lim_{x\to 3} \frac{[2x^2+4] - 22}{x-3}$$

$$= \lim_{x\to 3} \frac{2x^2 - 18}{x-3}$$

$\boxed{B}$

$$\lim_{x\to 0}\frac{\sin \pi x}{\tan 6x} = \lim_{x\to 0}\frac{\sin \pi x}{\dfrac{\sin 6x}{\cos 6x}}$$

$$= \lim_{x\to 0}\frac{\sin \pi x}{\sin 6x}(\cos 6x)$$

$$= \lim_{x\to 0}\underbrace{\frac{\sin \pi x}{\pi x}}_{1}\;\underbrace{\frac{6x}{\sin 6x}}_{1}\;\underbrace{(\cos 6x)}_{1}\;\underbrace{\left(\frac{\pi \cancel{x}}{6\cancel{x}}\right)}_{\frac{\pi}{6}}$$

$$= \frac{\pi}{6}$$

(7) The tangent line to $y = \left(2+\sqrt{x}\right)^2$ at $x = 1$ intersects the y axis where?

A. $(0,6)$ B. $(0,4)$ C. $(0,2)$ D. $(0,1)$ E. $(0,3)$

$y = (2+x^{\frac{1}{2}})^2 \Rightarrow y' = 2(2+x^{1/2})(0+\frac{1}{2}x^{-1/2})$

$y' = \cancel{2}(2+\sqrt{x})\left(\frac{1}{\cancel{2}\sqrt{x}}\right)$

$m_{tan} = y'(1) = (2+1)(\frac{1}{1}) = 3$

point on tangent line is $(1, y(1)) = (1, 9)$

$\therefore$ Eqn of tangent line

$y - 9 = 3(x-1) \Rightarrow y = 3x+6$

intersects y-axis when $x = 0$

$\therefore y = 6$ so $(0,6)$ $\boxed{A}$

(11) Given the following data

$f(0) = -3$	$f(1) = 4$
$f'(0) = 2$	$f'(1) = 3$
$g(0) = 1$	$g(1) = 0$
$g'(0) = -1$	$g'(1) = -2$

if $h(x) = \dfrac{f(g(x))}{g(x)}$, find $h'(0)$.

A. 7 **B.** 2 **C.** 4 **D.** 3 **E.** 1

$$h'(x) = \frac{g(x)\left[f'(g(x))g'(x)\right] - f(g(x))\left[g'(x)\right]}{g(x)^2}$$

$$\therefore\ h'(0) = \frac{g(0)\left[f'(g(0))g'(0)\right] - f(g(0))\left[g'(0)\right]}{g(0)^2}$$

$$= \frac{g(0)\left[f'(1)g'(0)\right] - f(1)\left[g'(0)\right]}{g(0)^2}$$

$$= \frac{(1)\left[(3)(-1)\right] - (4)\left[-1\right]}{1^2} = 1 \qquad \boxed{E}$$

(15) Use Implicit Differentiation to find y' if $x^2 + 5xy - 6y^4 = 18$

A. $\dfrac{2x+5y}{24y^3-5x}$ B. $\dfrac{2x+5y}{24y^3}$ C. $\dfrac{2x}{24y^3-5x}$

D. $\dfrac{2x+5y-18}{24y^3}$ E. $\dfrac{2x+5y-18}{24y^3-5x}$

$y = y(x)$

$$\frac{d}{dx}\{x^2+5xy-6y^4\} = \frac{d}{dx}\{18\}$$

$$\Rightarrow 2x+5\left[x\frac{dy}{dx}+y(1)\right]-24y^3\frac{dy}{dx}=0$$

$$\Rightarrow \frac{dy}{dx}\{5x-24y^3\} = -2x-5y$$

$$\frac{dy}{dx} = \frac{-2x-5y}{5x-24y^3} = \frac{-(2x+5y)}{-(-5x+24y^3)}$$

$$= \frac{2x+5y}{24y^3-5x} \quad \boxed{A}$$

(16) Use Logarithmic Differentiation to find the derivative of $f(x) = x^{\sin x}$

A. $(\sin x)\, x^{\sin x - 1}$ B. $x^{\sin x}(\cos x)(\ln x)$

C. $\dfrac{\sin x}{x} + (\cos x)(\ln x)$ D. $x^{\sin x}\left[\dfrac{\sin x}{x} + (\cos x)(\ln x)\right]$

E. $x \cos x + \sin x$

$$\ln f(x) = \ln\{x^{\sin x}\}$$

$$\ln f(x) = (\sin x)(\ln x)$$

$$\therefore \frac{d}{dx}\{\ln f(x)\} = \frac{d}{dx}\{(\sin x)(\ln x)\}$$

$$\frac{1}{f(x)} f'(x) = (\sin x)\left[\frac{1}{x}\right] + (\ln x)[\cos x]$$

$$\Rightarrow f'(x) = f(x)\left[\frac{\sin x}{x} + (\ln x)(\cos x)\right]$$

$$= x^{\sin x}\left[\frac{\sin x}{x} + (\ln x)(\cos x)\right]$$

$$\boxed{D}$$

Find $\frac{dy}{dx}$ if $\boxed{ye^{y^2} = 10x}$ $\boxed{2}$

<u>Implicit Diff:</u> $\frac{d}{dx}\{ye^{y^2}\} = \frac{d}{dx}\{10x\}$

$$y\frac{d}{dx}\{e^{y^2}\} + e^{y^2}\frac{d\{y\}}{dx} = 10$$

$$y\left[e^{y^2}\left(2y\frac{dy}{dx}\right)\right] + e^{y^2}\frac{dy}{dx} = 10$$

$$\frac{dy}{dx}\left(2y^2e^{y^2} + e^{y^2}\right) = 10$$

$$\frac{dy}{dx} = \frac{10}{e^{y^2}(2y^2+1)} \checkmark$$

Since $ye^{y^2} = 10x \Rightarrow e^{y^2} = \frac{10x}{y}$

$$\therefore \frac{dy}{dx} = \frac{10}{\frac{10x}{y}(2y^2+1)} = \frac{y}{x(2y^2+1)} \checkmark$$

(23) If $y = y(x)$ is defined implicitly by the equation $y\,e^{y^2} = 10x$, then $\frac{dy}{dx} =$

A. $y + 2ye^{y^2}$ B. $2y^2e^{y^2} + e^{y^2}$ C. $\frac{y}{x(2y^2+1)}$ D. $\frac{y}{(2y^2+1)}$

E. $\frac{y}{10(2y^2+1)}$

Log. Diff Method: $\ln(ye^{y^2}) = \ln(10x)$

$\Rightarrow \ln y + \ln(e^{y^2}) = \ln(10x)$

$\Rightarrow \ln y + y^2 = \ln(10x)$ now diff. w.r.t. x

$\Rightarrow \frac{1}{y}\frac{dy}{dx} + 2y\frac{dy}{dx} = \frac{1}{x} \Rightarrow \frac{dy}{dx} = \frac{\frac{1}{x}}{\frac{1}{y}+2y}$

$\Rightarrow \frac{dy}{dx} = \frac{\frac{1}{x}}{\frac{1+2y^2}{y}} = \frac{y}{x(1+2y^2)}$ $\boxed{C}$

Note: Could just do by Implicit Diff. also.

(22) Compute y'' if $x^2 + y^2 = 2y + 5$. Implicit Diff.

A. $\dfrac{d^2y}{dx^2} = \dfrac{1}{1-y} + \dfrac{x^2}{(1-y)^2}$ B. $\dfrac{d^2y}{dx^2} = \dfrac{1}{1-y} + \dfrac{x^2}{(1-y)^3}$

C. $\dfrac{d^2y}{dx^2} = \dfrac{1+x^2}{(1-y)^2}$ D. $\dfrac{d^2y}{dx^2} = \dfrac{1+x^2}{(1-y)^3}$

E. $\dfrac{d^2y}{dx^2} = \dfrac{x^2}{1-y} - \dfrac{1}{(1-y)^3}$

$y = y(x)$

$$\frac{d}{dx}\{x^2+y^2\} = \frac{d}{dx}\{2y+5\}$$

$$2x + 2y\frac{dy}{dx} = 2\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{-2x}{2y-2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-x}{y-1} \Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{-x}{y-1}\right)$$

$$= \frac{(y-1)(-1) - (-x)\left(\frac{dy}{dx}\right)}{(y-1)^2} = \frac{-(y-1) + x\left(\frac{-x}{y-1}\right)}{(y-1)^2}$$

$$= \frac{-(y-1)}{(y-1)^2} - \frac{x^2}{(y-1)^3} = \frac{-1}{y-1} - \frac{x^2}{(y-1)^3}$$

$$= \frac{1}{1-y} + \frac{x^2}{(1-y)^3} \quad \boxed{B}$$

(4) Which of these statements are **TRUE**?

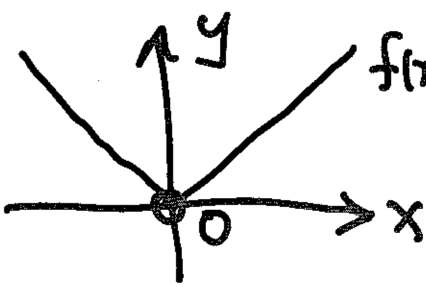
(i) If $f(x) = |x|$, then $f'(0)$ **DNE**

(ii) If $g(x) = \dfrac{|x|}{x}$, then $g'(0)$ **DNE**

(iii) If $h(x) = x|x|$, then $h'(0) = 0$

A. Only (i) and (ii) **B.** Only (i) **C.** Only (i) and (iii)

D. None are True **E.** All are True



$f(x) = |x|$ not diff. at $x=0$ (corner)

$\Rightarrow$ (i) TRUE

$$g(x) = \frac{|x|}{x} = \begin{cases} 1, & \text{if } x \geq 0 \\ -1, & \text{if } x < 0 \end{cases}$$

$\Rightarrow g(x)$ not cont at $x=0$

$\therefore g(x)$ not diff at $x=0$

$\therefore$ (ii) TRUE

$$h(x) = x|x| = \begin{cases} x^2, & \text{if } x \geq 0 \\ -x^2, & \text{if } x < 0 \end{cases}$$

$$\lim_{x \to 0^+} \frac{h(x) - h(0)}{x - 0} = \lim_{x \to 0^+} x = 0$$

$$\lim_{x \to 0^-} \frac{h(x) - h(0)}{x - 0} = \lim_{x \to 0^-} -x = 0$$

$$\therefore \lim_{x \to 0} \frac{h(x) - h(0)}{x - 0} = h'(0) = 0$$

$\therefore$ (iii) TRUE

$\boxed{E}$