Last Update: September 15

# Study Guide - Exam # 1

## I Review of Functions

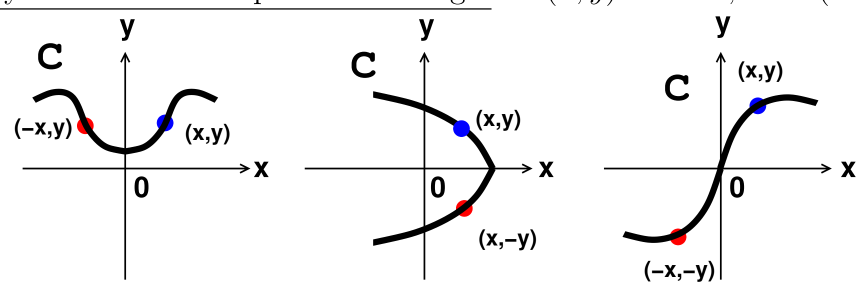
**(1)** A *function* $f$ assigns to a number $x$ a unique number $f(x)$; the set of all admissible $x$ is the *Domain* of $f$ and the set of all possible values $f(x)$ is the *Range* of $f$. The domain of $f(x)+g(x)$ and $f(x)g(x)$ is the intersection of the domains of $f$ and $g$. The domain of the quotient $\dfrac{f(x)}{g(x)}$ is the intersection of the domains of $f$ and $g$ where $g(x) \neq 0$.

**(2)** A curve $C$ represents the graph of a function if $C$ passes the Vertical Line Test (VLT). A function is one-to-one (hence has an inverse) if its graph passes the Horizontal Line Test (HLT).

**(3)** The composite function $\big(f \circ g\big)(x) = f\big(g(x)\big)$ is defined when the range of $g(x)$ is contained in the domain of $f$. Note that in general $\big(f \circ g\big)(x) \neq \big(g \circ f\big)(x)$.

**(4)** Symmetry in Curves $C$:

- $C$ is symmetric with respect to $y$ axis: If $(x, y)$ is on $C$, then $(-x, y)$ is also on $C$.
- $C$ is symmetric with respect to $x$ axis: If $(x, y)$ is on $C$, then $(x, -y)$ is also on $C$.
- $C$ is symmetric with respect to the origin: If $(x, y)$ is on $C$, then $(-x, -y)$ is also on $C$.




**(5)** If $f(-x) = f(x)$, then $f$ is an even function and its graph is symmetric w.r.t. the $y$ axis.
If $f(-x) = -f(x)$, then $f$ is an odd function and its graph is symmetric w.r.t. the origin.

**(6)** The domain of all polynomials $p(x) = c_n x^n + c_{n-1}x^{n-1} + \cdots + c_1 x + c_0$ is the set of all real numbers $x$ i.e., $-\infty < x < \infty$ or, equivalently, $(-\infty, \infty)$ .

A *rational function* is the quotient of polynomials: $f(x) = \dfrac{p(x)}{q(x)}$ and has domain the set of all real numbers $x$ for which the dominator polynomial $q(x) \neq 0$.

**(7)** Transformations of Functions $y = f(x)$ (useful for graphing):

(a) $\boxed{y = f(x) + c}$ vertical shift of c units

(b) $\boxed{y = f(x - c)}$ horizontal shift of c units

(c) $\boxed{y = cf(x)}$ vertical scaling: if $|c| > 1 \implies$ stretches; if $|c| < 1 \implies$ compresses

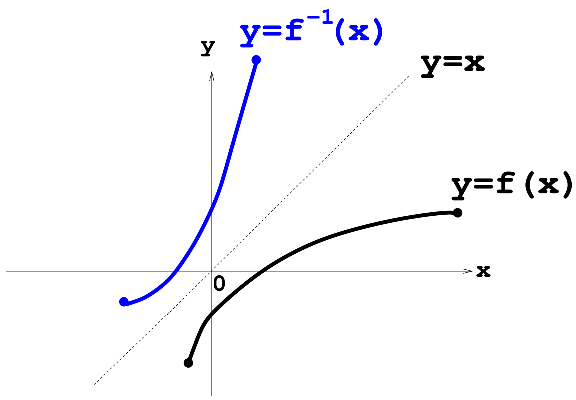
(d) $\boxed{y = f(cx)}$ horizontal scaling: if $|c| > 1 \implies$ compresses; if $|c| < 1 \implies$ stretches

**(8)** The *inverse* of a function $f$, denoted by $f^{-1}$ is defined by $\boxed{y = f^{-1}(x) \iff f(y) = x}$. A function $y = f(x)$ has an inverse if it is one-to-one i.e., its graph passes the *Horizontal Line Test* (HLT). Sometimes only a part of the graph passes the HLT, so that part of the function has an inverse. Note that $f^{-1}\big(f(x)\big) = x$ and also $f\big(f^{-1}(y)\big) = y$.

<ins>How to Find Inverse of $y = f(x)$</ins>

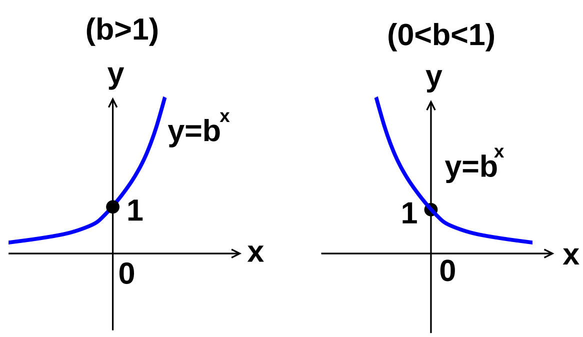
- Solve the equation $y = f(x)$ for $x$, to get $x = f^{-1}(y)$.
- Interchange $x$ and $y$ in equation $x = f^{-1}(y)$ to obtain the answer $y = f^{-1}(x)$.

To get the graph of $y = f^{-1}(x)$, reflect the graph of $y = f(x)$ across the line $y = x$ :




**(9)** Exponential and Logarithmic Functions

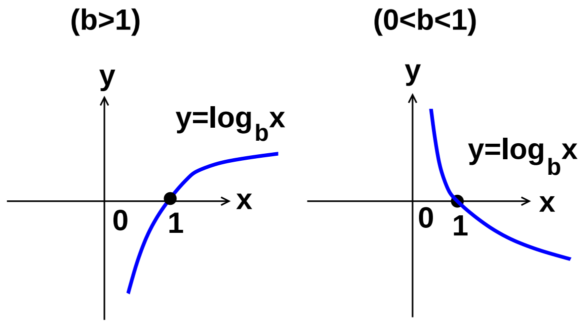
(a) If $b > 0$ and $b \neq 1$, the *Exponential Function* is $y = f(x) = b^x$. Domain of all exponential functions is the set of all real numbers $x$ and the range is the set of all positive numbers :




(b) <ins>Law of Exponents:</ins>

- $b^{x+y} = b^x\, b^y$
- $b^{x-y} = \dfrac{b^x}{b^y}$
- $\big(b^x\big)^r = b^{xr}$

(c) If $b > 0$ and $b \neq 1$, the *Logarithm Function* with base $b$ is defined by: $\boxed{y = \log_b x \iff b^y = x}$
The domain of $y = \log_b x$ is all $x > 0$ while the range is all real numbers.
($\log_e x = \ln x$ is the Natural Logarithm Function where $e = 2.718\cdots$)




(d) Inverse Relations: (let $b > 0$ and $b \neq 1$)

- $b^{\log_b x} = x$, for $x > 0$
- $\log_b \left(b^x\right) = x$, for all real numbers $x$

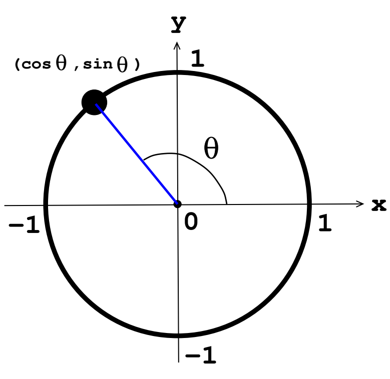
(e) Law of Logarithms:

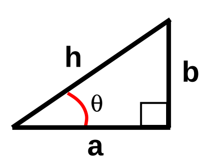
- $\log_b \left(xy\right) = \log_b x + \log_b y$
- $\log_b \left(\dfrac{x}{y}\right) = \log_b x - \log_b y$
- $\log_b \left(x^p\right) = p \log_b x$

(f) Change of Base in Logarithms: $\boxed{\log_b x = \dfrac{\ln x}{\ln b}}$

**(10)** Trigonometric Functions

(a) The point $(\cos\theta, \sin\theta)$ is the point on the circle of radius 1:




| $\theta$ | | $\sin\theta$ | $\cos\theta$ |
|---|---|---|---|
| (0°) | 0 | 0 | 1 |
| (30°) | $\frac{\pi}{6}$ | $\frac{1}{2}$ | $\frac{\sqrt{3}}{2}$ |
| (45°) | $\frac{\pi}{4}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{2}}{2}$ |
| (60°) | $\frac{\pi}{3}$ | $\frac{\sqrt{3}}{2}$ | $\frac{1}{2}$ |
| (90°) | $\frac{\pi}{2}$ | 1 | 0 |

(b) Basic definitions:

$$\boxed{\sin\theta = \frac{b}{h}; \quad \cos\theta = \frac{a}{h}; \quad \tan\theta = \frac{b}{a} = \frac{\sin\theta}{\cos\theta}}$$

Also $\sec\theta = \dfrac{1}{\cos\theta}$; $\csc\theta = \dfrac{1}{\sin\theta}$; $\cot\theta = \dfrac{1}{\tan\theta}$

.

(c) Recall, $\begin{cases} \sin(\theta+2\pi)=\sin\theta \\ \cos(\theta+2\pi)=\cos\theta \\ \tan(\theta+\pi)=\tan\theta \end{cases}$ and also $\begin{cases} \cos(-\theta)=\cos\theta & \big[\text{ cosine is an } \underline{\textbf{even}} \text{ function } \big] \\ \\ \sin(-\theta)=-\sin\theta & \big[\text{ sine is an } \underline{\textbf{odd}} \text{ function } \big] \end{cases}$

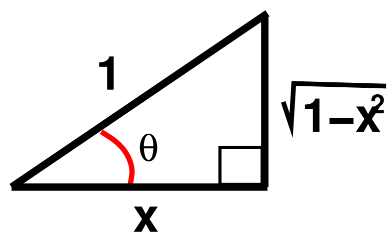
(d) Basic Identities

- $\sin^2\theta+\cos^2\theta=1$
- $\tan^2\theta+1=\sec^2\theta$ and $1+\cot^2\theta=\csc^2\theta$
- $\cos 2\theta=\cos^2\theta-\sin^2\theta$ and $\sin 2\theta=2\sin\theta\cos\theta$
- $\cos^2\theta=\dfrac{1+\cos 2\theta}{2}$ and $\sin^2\theta=\dfrac{1-\cos 2\theta}{2}$

(e) Inverse Trig Functions

- $y=\sin^{-1}x$, where $-\dfrac{\pi}{2}\le\sin^{-1}x\le\dfrac{\pi}{2}$
- $y=\cos^{-1}x$, where $0\le\cos^{-1}x\le\pi$
- $y=\tan^{-1}x$, where $-\dfrac{\pi}{2}<\tan^{-1}x<\dfrac{\pi}{2}$

(f) How to simplify expressions like $\sin\big(\cos^{-1}x\big)$. For this example, let $\theta=\cos^{-1}x$ and hence $\cos\theta=x=\dfrac{x}{1}$, now draw the corresponding right triangle:




(h) Transformations of Trig Functions: For example, $y=A\Big(\sin B(\theta-C)\Big)+D$, then

$|A|=\text{amplitude}$, $\dfrac{2\pi}{|B|}=\text{period}$, horizontal shift of $C$ units; vertical shift of $D$ units

## $\boxed{\text{II}}$ **Limits**:

(1) Average Velocity over $\big[a,b\big]$ is $v_{av}=\dfrac{s(b)-s(a)}{b-a}$; instantaneous velocity at $t=a$ is $v_{ins}=\lim\limits_{b\to a}\dfrac{s(b)-s(a)}{b-a}$. Slope of secant line through $\big(a,f(a)\big)$ and $\big(b,f(b)\big)$ is $m_{\text{sec}}=\dfrac{f(b)-f(a)}{b-a}$; slope of tangent line to curve $y=f(x)$ at $x=a$ is $m_{\tan}=\lim\limits_{b\to a}\dfrac{f(b)-f(a)}{b-a}$

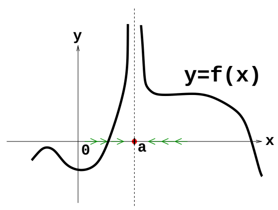
(2) Meaning of $\lim\limits_{x\to a}f(x)=L$; meaning of one-sided limits $\lim\limits_{x\to a^+}f(x)=L$ and $\lim\limits_{x\to a^-}f(x)=L$

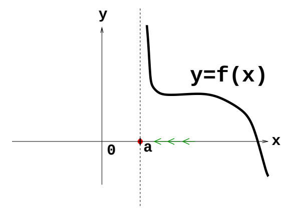
**Theorem:** $\lim\limits_{x\to a}f(x)=L \iff \lim\limits_{x\to a^-}f(x)=\lim\limits_{x\to a^+}f(x)=L$

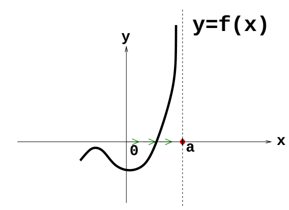
(3) **Limit Laws**; using these to compute limits.

(4) **Squeeze/Sandwich Theorem:** If $g_1(x) \leq f(x) \leq g_2(x)$ with both $\lim_{x\to a} g_1(x) = L$ and $\lim_{x\to a} g_2(x) = L$, then $\lim_{x\to a} f(x) = L$

(5) **Infinite Limits**: meaning of $\lim_{x\to a} f(x) = \infty$ (or $-\infty$); one-sided infinite limits $\lim_{x\to a^+} f(x) = \infty$ (or $-\infty$); $\lim_{x\to a^-} f(x) = \infty$ (or $-\infty$);



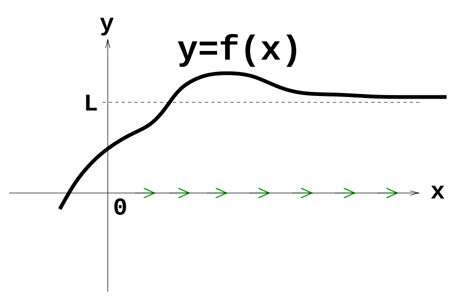

(6) The line $x = a$ is a *Vertical Asymptote* of $f$ if any of these 3 limits is infinite ($\infty$ or $-\infty$):

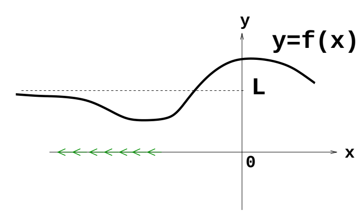
$$\lim_{x\to a} f(x)\,, \quad \lim_{x\to a^+} f(x)\,, \quad \lim_{x\to a^-} f(x)$$

Remark For a rational function $f(x) = \dfrac{p(x)}{q(x)}$, the vertical asymptotes are *among* the zeros of the polynomial $q(x)$ in the denominator. Verify which are actually vertical asymptotes of $f$.

(7) **Limits at Infinity**

(a) Meaning of $\lim_{x\to\infty} f(x) = L$ and $\lim_{x\to-\infty} f(x) = L$




Remark: The line $y = L$ is a *Horizontal Asymptote* of $f(x)$.

(b) *Infinite Limits at Infinity* : $\lim_{x\to\infty} f(x) = \infty$ (or $-\infty$) and $\lim_{x\to-\infty} f(x) = \infty$ (or $-\infty$)

(c) **Limits at $\infty$ of Powers**

(i) $\lim_{x\to\pm\infty} x^n = \infty$, when $n$ is **even**

(ii) $\lim_{x\to\infty} x^n = \infty$ and $\lim_{x\to-\infty} x^n = -\infty$, when $n$ is **odd**

(iii) $\lim_{x\to\pm\infty} \dfrac{1}{x^n} = 0$

(iv) If $p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ is a polynomial, then $\lim_{x\to\pm\infty} p(x) = \lim_{x\to\pm\infty} a_n x^n = \pm\infty$, depends on $a_n$ and also the degree of $p(x)$

(d) The **End Behavior** of a polynomial or rational function is their behaviors as $x \to \pm\infty$.

**Theorem - End Behavior and Asymptotes of Rational Functions**

Suppose $f(x) = \dfrac{p(x)}{q(x)}$, where $p(x) = a_m x^m + a_{m-1}x^{m-1} + \cdots + a_1 x + a_0$ and $q(x) = b_n x^n + b_{n-1}x^{n-1} + \cdots + b_1 x + b_0$, where $a_m \neq 0, b_n \neq 0$

(i) If $\deg p(x) < \deg q(x) \implies \lim\limits_{x\to\pm\infty} f(x) = 0$ $\quad \left[\, y = 0 \text{ is horizontal asymp. of } f \right]$

(ii) If $\deg p(x) = \deg q(x) \implies \lim\limits_{x\to\pm\infty} f(x) = \dfrac{a_m}{b_n}$ $\quad \left[\, y = \dfrac{a_m}{b_n} \text{ is horizontal asymp. of } f \right]$

(iii) If $\deg p(x) > \deg q(x) \implies \lim\limits_{x\to\pm\infty} f(x) = \infty$ (or $-\infty$) $\left[ f \text{ has } \underline{\text{NO}} \text{ horizontal asymp.} \right]$

(iv) If $\deg p(x) = \deg q(x) + 1 \implies \lim\limits_{x\to\pm\infty} f(x) = \infty$ or $(-\infty)$ $\Big[\ f$ has a **Slant/Oblique** asymptote and NO horizontal asymp.; to find it perform long division on $\dfrac{p(x)}{q(x)}$ $\Big]$

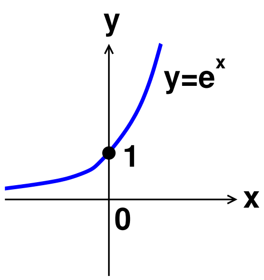
(v) If $f(x) = \dfrac{p(x)}{q(x)}$ is in reduced form (i.e., $p(x)$ and $q(x)$ have no common factors), then the vertical asymptotes of $f$ are precisely the zeros of $q(x)$.

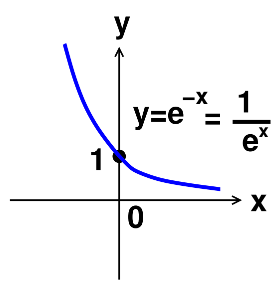
**Theorem - End Behavior of** $e^x, e^{-x}, \ln x$

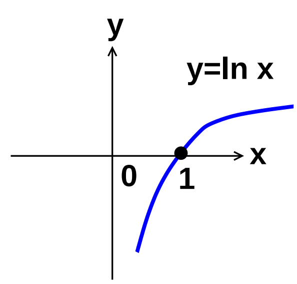
$$\lim_{x\to\infty} e^x = \infty \qquad \lim_{x\to-\infty} e^x = 0$$

$$\lim_{x\to\infty} e^{-x} = \lim_{x\to\infty} \frac{1}{e^x} = 0 \qquad \lim_{x\to-\infty} e^{-x} = \lim_{x\to-\infty} \frac{1}{e^x} = \infty$$

$$\lim_{x\to 0^+} \ln x = -\infty \qquad \lim_{x\to\infty} \ln x = \infty$$




Remark: $\lim\limits_{x\to\infty} \tan^{-1} x = \dfrac{\pi}{2}$ and $\lim\limits_{x\to-\infty} \tan^{-1} x = -\dfrac{\pi}{2}$

## $\boxed{\text{III}}$ **Continuity**:

(1) $f(x)$ is **continuous** at $x = a$ if $\boxed{\lim_{x \to a} f(x) = f(a)}$

Continuity Checklist for $f(x)$ to be continuous at $a$ :

$\boxed{1}$ $f(a)$ is defined

$\boxed{2}$ $\lim_{x \to a} f(x)$ exists (and is finite)

$\boxed{3}$ $\lim_{x \to a} f(x) = f(a)$

(2) Types of discontinuities: *Jump discontinuity, Infinite discontinuity, Removable discontinuity.*

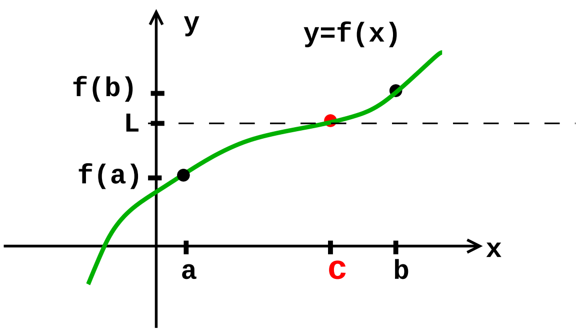
(3) Left and right continuity; Continuity Rules; continuity of polynomials and rational functions; continuity of composite functions; continuity of inverse functions; continuity of inverse trig functions. Piece-wise continuous function.

(4) **Theorem** Limits of Composite Functions:

(a) If $g$ is continuous at $a$ and $f$ continuous at $g(a)$, then $\lim_{x \to a} f\Big(g(x)\Big) = f\left(\lim_{x \to a} g(x)\right)$

(b) If $\lim_{x \to a} g(x) = L$ and $f$ continuous at $L$, then $\lim_{x \to a} f\Big(g(x)\Big) = f\left(\lim_{x \to a} g(x)\right) = f(L)$

(5) **Intermediate Value Theorem** - If $f$ is continuous on interval $\big[a, b\big]$ and $L$ is a number strickly between $f(a)$ and $f(b)$, then there is at least one number $c$ in the interval $\big(a, b\big)$ such that $f(c) = L$:




## $\boxed{\text{IV}}$ **Derivatives**:

(1) The *derivative* of $f$ at a point $a$: $f'(a) = \lim_{x \to a} \dfrac{f(x) - f(a)}{x - a}$ or $f'(a) = \lim_{h \to 0} \dfrac{f(a+h) - f(a)}{h}$

(2) Physical interpretation of derivative:

$$f'(a) = \begin{cases} \text{slope of tangent line the graph of } y = f(x) \textit{ at } a \\ \text{velocity at time } a \\ \text{(instantaneous) rate of change of } f \text{ at } a \end{cases}$$

# Practice Problems - Exam #1

**1.** The domain of $f(x) = \sqrt{x-1} - \dfrac{1}{\sqrt{2-x}}$ is

A. $\big[1, 2\big)$ B. $\big(1, 2\big)$ C. $\big(1, 2\big]$ D. $\big[1, \infty\big)$ E. $\big[1, 2\big]$

**2.** Find the domain of $g(x) = \dfrac{1}{\ln\sqrt{x-10}}$

A. $\big(10,\ \infty\big)$ B. $\big[10,\ \infty\big)$ C. $\big(e^{11},\ \infty\big)$ D. $\big(10,\ 11\big) \cup \big(11,\ \infty\big)$
E. $\big[10,\ 11\big) \cup \big(11,\ \infty\big)$

**3.** If $f(x) = \dfrac{4}{x-1}$ and $g(x) = 2x$, for what value(s) of $x$ is $(f \circ g)(x) = (g \circ f)(x)$ ?

A. $x = \frac{1}{3}$ only B. $x = 2$ only C. $x = 3$ only D. $x = -1$ or $x = 2$
E. $x = \frac{1}{3}$ or $x = 2$

**4.** Find the inverse of the function $f(x) = \dfrac{6x-1}{2x+9}$

A. $\dfrac{-9x-1}{2x-6}$ B. $\dfrac{9x+1}{2x+6}$ C. $\dfrac{2x+9}{6x-1}$ D. $\dfrac{9x+1}{2x-6}$ E. $\ln(6x-1) - \ln(2x-6)$

**5.** If $f(x) = 3 + \sqrt{2+7x}$, find $f^{-1}(5)$

A. $\dfrac{1}{7}$ B. $\dfrac{2}{7}$ C. $\dfrac{3}{7}$ D. $\dfrac{4}{7}$ E. $\dfrac{5}{7}$

**6.** Which of the following does **NOT** have an inverse function?

A. $y = \sin x,\ -\frac{\pi}{2} \le x \le \frac{\pi}{2}$ B. $y = x^3 + 2$ C. $y = \dfrac{x}{x^2+1}$ D. $y = \frac{1}{2}e^x$

E. $y = \ln(x-2),\ x > 2$

**7.** The graph of a function $g$ is obtained from the graph if $f$ by first compressing vertically by a factor of 3, then shifting to the right by 2units, and then shifting up by 1 unit. What is $g(x) =$?

A. $f\left(\frac{x}{3} + 1\right) + 2$ B. $f\left(\frac{x+3}{3} + 1\right)$ C. $\frac{1}{3}f(x-2) + 1$ D. $3f(x+2) - 1$
E. $f(3(x-2)) - 1$

**8.** Solve $\ln(x^2 - 9) - \ln(x-3) = 2$

A. $e^2 - 3$ B. $e^2 + 3$ C. $\dfrac{1}{e^2} - 3$ D. $\dfrac{1}{e^2} + 3$ E. $e + 3$

**9.** Find all values of $x$ in the interval $\big[0, 2\pi\big]$ satisfying $2\sin x\cos x + \cos x = 0$

A. $\frac{\pi}{2}, \frac{3\pi}{2}$ B. $\frac{7\pi}{6}, \frac{11\pi}{6}$ C. $\frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$ D. $\frac{2\pi}{3}, \frac{4\pi}{3}$ E. No such values in given interval

**10.** The altitude, in feet, of a drone $t$ seconds after it takes off is given by $h(t) = \dfrac{t^3}{6} + 4$. What is the drone's average velocity in feet per second over the time interval $\big[0,\ 3\big]$?

A. 1 B. $\frac{3}{2}$ C. $\frac{9}{2}$ D. 9 E. 27

**11.** Evaluate $\sin^{-1}\left(\sin\dfrac{4\pi}{3}\right)$

A. $\dfrac{2\pi}{3}$ B. $-\dfrac{\pi}{3}$ C. $\dfrac{4\pi}{3}$ D. $-\dfrac{2\pi}{3}$ E. $\dfrac{\pi}{3}$

**12.** Write the trigonometric expression $\sin\left(\tan^{-1} u\right)$ as an algebraic expression in $u$.

A. $\dfrac{1}{\sqrt{1+u^2}}$ B. $\dfrac{u}{\sqrt{1+u^2}}$ C. $\dfrac{1}{\sqrt{u^2-1}}$ D. $\dfrac{u}{\sqrt{1-u^2}}$ E. $\dfrac{u}{\sqrt{u^2-1}}$

**13.** Find the limit: $\displaystyle\lim_{x\to 1} \frac{x-1}{\sqrt{2x+23}-5}$

A. $\dfrac{5}{2}$ B. 5 C. $\dfrac{1}{2}$ D. 10 E. $\dfrac{2}{5}$

**14.** Find the limit: $\displaystyle\lim_{x\to 0} x^2 \cos\left(\frac{2}{x}\right)$

A. 1 B. 0 C. $-\infty$ D. $\infty$ E. **DNE**

**15.** Find the limit: $\displaystyle\lim_{t\to -2^+} \frac{t-1}{\sqrt{(t+3)(t+2)}}$

A. $\infty$ B. $-3$ C. 3 D. **DNE** and is neither $\infty$ nor $-\infty$ E. $-\infty$

**16.** Find the limit: $\displaystyle\lim_{t\to 2^-} \frac{|2t-4|}{t^2-4}$

A. $\infty$ B. $-\dfrac{1}{2}$ C. $\dfrac{1}{2}$ D. **DNE** and is neither $\infty$ nor $-\infty$ E. $-\infty$

**17.** Find all vertical asymptotes of $f(x) = \dfrac{-x^2+16}{x^2+5x+4}$

A. $x=-1$ B. $x=-1,\ x=4$ C. $x=1,\ x=-4$ D. $x=-1,\ x=-4$ E. $x=4$

**18.** Suppose $f(x) = \dfrac{2x^3+16x^2+30x}{x^3+5x^2}$. Which of the following statements are **correct**?

(i) $y=2$ is a horizontal asymptote
(ii) $x=-5$ is a vertical asymptote
(iii) $\displaystyle\lim_{x\to 0} f(x) = \infty$

A. Only statement (i) B. Only statements (i) and (ii) C. Only statements (i) and (iii) D. All three are correct E. Only statements (ii) and (iii)

**19.** The line $y=-3$ is the LEFT horizontal asymptote (asymptote as $x\to-\infty$) of which function?

A. $y=-3e^x$ B. $y=\dfrac{1}{x+3}$ C. $y=-\dfrac{3x}{1+x^2}$ D. $y=\dfrac{12x^2-x}{1+4x^2}$

E. $y=\dfrac{e^x-3}{e^x+1}$

**20.** The function $y=f(x)=\dfrac{2x^3-1}{x^2-5x+5}$ has a slant (oblique) asymptote given by

A. $y=2x+10$ B. $y=2x-5$ C. $y=2x-1$ D. $y=2x+5$ E. $y=2x$

**21.** Find the value of $c$ such that $f$ is continuous at $x=2$: $f(x) = \begin{cases} \dfrac{x^2-5x+c}{x-2}, & \text{if } x<2 \\[2ex] \tan\left(\dfrac{3\pi}{2x}\right), & \text{if } x\geq 2 \end{cases}$

A. $c=6$ B. No value of $c$ C. $c=3$ D. $c=-1$ E. $c=4$

**22.** Determine which statements about $f$ are **True** and which are **False** if

$$f(x) = \begin{cases} 1, & \text{if } x \leq -1 \\ x, & \text{if } -1 < x < 1 \\ 1, & \text{if } x \geq 1 \end{cases}$$

(I) $f$ is discontinuous at $x = 1$

(II) $f$ is continuous from the left at $x = -1$

(III) $f$ is continuous from the right at $x = -1$

A. (I) is True; (II) and (III) are False  B. (II) is True; (I) and (III) are False  C. (III) is True; (I) and (II) are False  D. (II) and (III) are True; (II) and (III) is False  E. (II) and (III) are True; (II) and (I) is False

**23.** $\displaystyle\lim_{x\to 1} \left( \frac{1-x}{1-\sqrt{x}} \right)^2 =$

A. 4  B. 1  C. $\frac{1}{2}$  D. $\frac{1}{\sqrt{2}}$  E. **DNE**

**24.** Which of the statements below are TRUE for the function $f(x) = \dfrac{4x^3 - 4x^2}{x(x-1)^2}$?

(i) $x = 0$ is a removable discontinuity

(ii) $x = 0$ and $x = 1$ are both removable discontinuities

(iii) $x = 1$ is an infinite discontinuity

A. (i) and (ii)  B. (ii) and (iii)  C. (i) and (iii)  D. None are true  E. All are true

**25.** If the tangent line to the graph if $y = f(x)$ at $x = 4$ goes through the points $(5, 3)$ and $(7, 7)$, what is $f(4)$?

A. $-2$  B. $-1$  C. 0  D. 1  E. 2

**26.** The quantity $\displaystyle\lim_{h\to 0} \frac{\sqrt{9+h}-3}{h}$ represents which of the following?

A. $f'(3)$ with $f(x) = \sqrt{x}$  B. $f'(6)$ with $f(x) = \sqrt{x}$  C. $f'(-6)$ with $f(x) = \sqrt{x+3}$  D. $f'(-3)$ with $f(x) = \sqrt{x+9}$  E. $f'(6)$ with $f(x) = \sqrt{x+3}$

**27.** If $f(x) = \dfrac{8e^x + 3e^{3x}}{2e^{2x} - e^{3x}}$, which of the following statements are TRUE?

(I) $\displaystyle\lim_{x\to\infty} f(x) = -3$

(II) $\displaystyle\lim_{x\to-\infty} f(x) = \infty$

(III) $f(x)$ has a vertical asymptote at $x = \ln 2$

A. Only (I)  B Only (I) and (II)  C. Only (I) and (III)  D. Only (II) and (III)  E. All three are true

**28.** $\displaystyle\lim_{\theta\to 0^-} \frac{|\sin\theta|}{\cos^2\theta - 1} =$

A. 1  B. 0  C. $\infty$  D. $-\infty$  E. **DNE**

## Answers

**1.** A **2.** D **3.** A **4.** A **5.** B **6.** C **7.** C **8.** A **9.** C **10.** B **11.** B
**12.** B **13.** B **14.** B **15.** E **16.** B **17.** A **18.** A **19.** E **20.** A **21.** A
**22.** B **23.** A **24.** C **25.** D **26.** E **27.** E **28.** D