(1) If $\boldsymbol{f(x) = 2x^2 + 4}$, which of the following is $\boldsymbol{f'(3)}$?

A. $\displaystyle \lim_{h\to 0} \frac{2(3+h)^2 - 10}{h}$ B. $\displaystyle \lim_{x\to 3} \frac{2x^2 - 18}{x-3}$

C. $\displaystyle \lim_{h\to 0} \frac{2(3+h)^2 + 18}{h}$ D. $\displaystyle \lim_{x\to 3} \frac{2x^2 + 18x}{x-3}$ E. $\displaystyle \lim_{h\to 0} \frac{2h^2 - 18}{h-3}$

$$f'(3) = \lim_{h\to 0} \frac{f(3+h) - f(3)}{h}$$

$$= \lim_{h\to 0} \frac{[2(3+h)^2+4] - [22]}{h}$$

$$= \lim_{h\to 0} \frac{2(3+h)^2 - 18}{h}$$

Try other form:

$$f'(3) = \lim_{x\to 3} \frac{f(x) - f(3)}{x-3} = \lim_{x\to 3} \frac{[2x^2+4] - 22}{x-3}$$

$$= \lim_{x\to 3} \frac{2x^2 - 18}{x-3}$$

$\boxed{B}$

(2) Find all values of x for which the graph of $f(x) = \dfrac{2x-1}{x+1}$ has a tangent line parallel to the line $y = 3x + 1$.

A. -2 and 0 **B.** 3 and 0 **C.** 0 and 2 **D.** -3 and 1

E. -3 and 0

$$f'(x) = \frac{(x+1)(2)-(2x-1)(1)}{(x+1)^2} = \frac{3}{(x+1)^2} = m_{tan}$$

Hence want x such that

$$\frac{3}{(x+1)^2} = 3 \Rightarrow \frac{1}{(x+1)^2} = 1$$

$$\Rightarrow (x+1)^2 = 1 \Rightarrow x+1 = \pm 1$$

so $x+1 = 1$ and $x+1 = -1$

$$\Downarrow \qquad\qquad \Downarrow$$

$x = 0$ $\qquad$ $x = -2$

$\boxed{A}$

(3) A particle is moving on a straight line so that its displacement from the origin is given by $\boldsymbol{s(t) = t^3 - 6t^2 + 12t - 8}$. During which of the following time intervals is the particle speeding up?

A. $\mathbf{(2,\ \infty)}$ only **B.** $\mathbf{(0,\ 3)}$ only **C.** $\mathbf{(0,\ \infty)}$ only
D. $\mathbf{(0, 1) \cup (2,\ \infty)}$ **E.** $\mathbf{(0,\ 2)}$ only

Particle is speeding up when acceleration is positive.

$\therefore$ want $a = s''(t) > 0$

$s'(t) = 3t^2 - 12t + 12$

$s''(t) = 6t - 12 = 6(t-2) > 0$ when $t > 2$

$\boxed{A}$

(4) Which of these statements are **TRUE**?

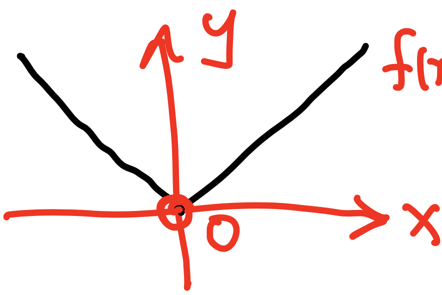
(i) If $\boldsymbol{f(x) = |x|}$, then $\boldsymbol{f'(0)}$ **DNE**

(ii) If $\boldsymbol{g(x) = \dfrac{|x|}{x}}$, then $\boldsymbol{g'(0)}$ **DNE**

(iii) If $\boldsymbol{h(x) = x|x|}$, then $\boldsymbol{h'(0) = 0}$

A. Only (i) and (ii) **B.** Only (i) **C.** Only (i) and (iii)

D. None are True **E.** All are True



$f(x)=|x|$ not diff. at $x=0$ (corner)

$\Rightarrow$ (i) TRUE

$$g(x)=\frac{|x|}{x}=\begin{cases}1, & \text{if } x\geq 0\\ -1, & \text{if } x<0\end{cases} \Rightarrow g(x) \text{ not cont at } x=0$$

$\therefore$ $g(x)$ not diff at $x=0$

$\therefore$ (ii) TRUE

$$h(x)=x|x|=\begin{cases}x^2, & \text{if } x\geq 0\\ -x^2, & \text{if } x<0\end{cases}$$

$$\lim_{x\to 0^+}\frac{h(x)-h(0)}{x-0}=\lim_{x\to 0^+} x=0$$

$$\lim_{x\to 0^-}\frac{h(x)-h(0)}{x-0}=\lim_{x\to 0^-} -x=0$$

$$\therefore \lim_{x\to 0}\frac{h(x)-h(0)}{x-0}=h'(0)=0$$

$\therefore$ (iii) TRUE

$\boxed{E}$

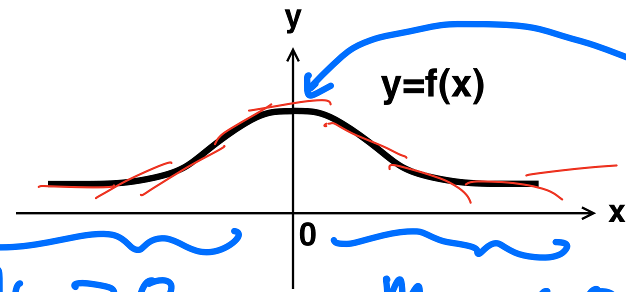
(5) If the normal line to the graph of $\boldsymbol{y = f(x)}$ at $\boldsymbol{(2, 6)}$ is $\boldsymbol{y = \frac{2}{3}x - 4}$, what is $\boldsymbol{f'(2)}$?

A. $\frac{3}{2}$ **B.** $\frac{2}{3}$ **C.** $-\frac{3}{2}$ **D.** $\mathbf{2}$ **E.** Cannot be determined

$y = \frac{2}{3}x - 4$ normal line has slope $m_{normal} = \frac{2}{3}$ but $m_{tan} = -\frac{1}{m_{normal}} = -\frac{3}{2}$

$\boxed{C}$

(6) Given the graph of $\boldsymbol{y} = \boldsymbol{f}(\boldsymbol{x})$ as shown here:

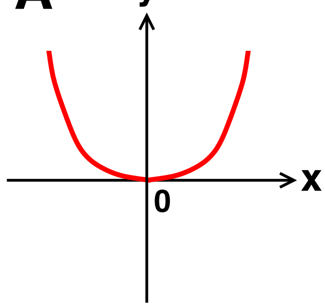


$m_{tan} = 0$

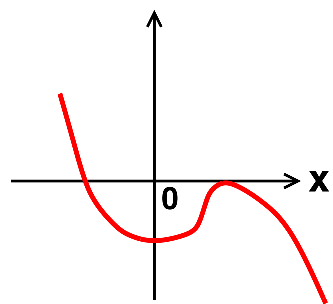
$m_{tan} > 0$ $m_{tan} < 0$

then which graph below looks most like graph of the derivative $\boldsymbol{f}'(\boldsymbol{x})$?

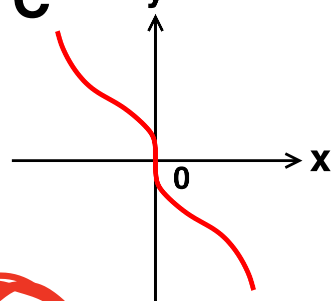
A



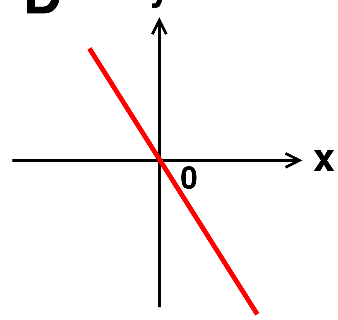
B



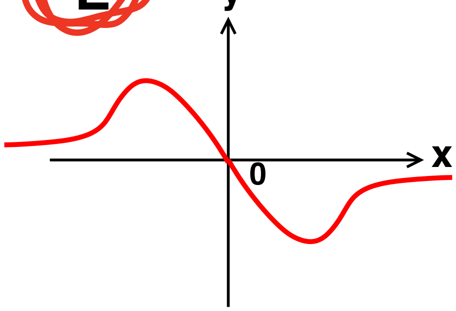
C



D



E



(7) The tangent line to $\boldsymbol{y} = \left(\mathbf{2} + \sqrt{\boldsymbol{x}}\right)^{\mathbf{2}}$ at $\boldsymbol{x} = \mathbf{1}$ intersects the $\boldsymbol{y}$ axis where?

A. $(\mathbf{0}, \mathbf{6})$ **B.** $(\mathbf{0}, \mathbf{4})$ **C.** $(\mathbf{0}, \mathbf{2})$ **D.** $(\mathbf{0}, \mathbf{1})$ **E.** $(\mathbf{0}, \mathbf{3})$

$$y = \left(2 + x^{\frac{1}{2}}\right)^2 \Rightarrow y' = 2\left(2 + x^{1/2}\right)\left(0 + \tfrac{1}{2}x^{-1/2}\right)$$

$$y' = \cancel{2}\left(2 + \sqrt{x}\right)\left(\frac{1}{\cancel{2}\sqrt{x}}\right)$$

$$m_{tan} = y'(1) = (2+1)\left(\frac{1}{1}\right) = 3$$

point on tangent line is $(1, y(1)) = (1, 9)$

$\therefore$ Eqn of tangent line

$$y - 9 = 3(x-1) \Rightarrow y = 3x + 6$$

intersects y-axis when $x = 0$

$\therefore$ $y = 6$ so $(0, 6)$

$\boxed{A}$

(8) $\lim_{x\to 0} \dfrac{\tan \pi x}{x \sec x} =$

A. $\dfrac{1}{\pi}$ B. 0 C. 1 D. π E. DNE

$$\lim_{x\to 0} \frac{\tan \pi x}{x \sec x} = \lim_{x\to 0} \frac{\frac{\sin \pi x}{\cos \pi x}}{x \, \frac{1}{\cos x}}$$

$$= \lim_{x\to 0} \frac{(\sin \pi x) \cos x}{x \cos \pi x}$$

$$= \lim_{x\to 0} \frac{\sin \pi x}{\pi x} \cdot \frac{1}{\cos \pi x} \cdot (\cos x)(\pi)$$

$$= (1)(1)(1)\pi = \pi$$

$\boxed{D}$

(9) Find the derivative of $f(x) = \dfrac{1}{\sqrt[3]{1-x^2}}$

A. $\dfrac{2x}{3(1-x^2)^{\frac{1}{3}}}$ B. $\dfrac{-2x}{3(1-x^2)^{\frac{4}{3}}}$ C. $\dfrac{-2x}{3(1-x^2)^{\frac{1}{3}}}$ D. $\dfrac{2x}{3(1-x^2)^{\frac{2}{3}}}$

E. $\dfrac{2x}{3(1-x^2)^{\frac{4}{3}}}$

$$f(x) = (1-x^2)^{-1/3}$$

$$\Rightarrow f'(x) = -\frac{1}{3}(1-x^2)^{-1/3-1}(-2x)$$

$$= -\frac{1}{3}(1-x^2)^{-4/3}(-2x)$$

$$= \frac{2x}{3(1-x^2)^{4/3}}$$

$\boxed{E}$

(10) If $\boldsymbol{y = \sqrt{x+\sqrt{x}}}$, then $\boldsymbol{y' =}$

A. $\dfrac{1+\sqrt{x}}{2\sqrt{x+\sqrt{x}}}$ **B.** $\dfrac{1+\sqrt{x}}{4\sqrt{x+\sqrt{x}}}$ **C.** $\dfrac{1+2\sqrt{x}}{2\sqrt{x+\sqrt{x}}}$

D. $\dfrac{1+2\sqrt{x}}{2\sqrt{x}\sqrt{x+\sqrt{x}}}$ **E.** $\dfrac{1+2\sqrt{x}}{4\sqrt{x}\sqrt{x+\sqrt{x}}}$

$$y = (x + x^{1/2})^{1/2}$$

$$\Rightarrow y' = \frac{1}{2}(x + x^{1/2})^{-1/2}\left(1 + \frac{1}{2}x^{-1/2}\right)$$

$$y' = \frac{1}{2\sqrt{x+\sqrt{x}}}\left(1 + \frac{1}{2\sqrt{x}}\right)$$

$$= \frac{1}{2\sqrt{x+\sqrt{x}}}\left(\frac{2\sqrt{x}+1}{2\sqrt{x}}\right)$$

$$= \frac{2\sqrt{x}+1}{4\sqrt{x}\left(\sqrt{x+\sqrt{x}}\right)} \quad \boxed{E}$$

(11) Given the following data

$f(0) = -3$	$f(1) = 4$
$f'(0) = 2$	$f'(1) = 3$
$g(0) = 1$	$g(1) = 0$
$g'(0) = -1$	$g'(1) = -2$

if $h(x) = \dfrac{f(g(x))}{g(x)}$, find $h'(0)$.

A. 7 B. 2 C. 4 D. 3 E. 1

$$h'(x) = \frac{g(x)\left[f'(g(x))g'(x)\right] - f(g(x))\left[g'(x)\right]}{g(x)^2}$$

$$\therefore h'(0) = \frac{g(0)\left[f'(g(0))g'(0)\right] - f(g(0))\left[g'(0)\right]}{g(0)^2}$$

$$= \frac{g(0)\left[f'(1)g'(0)\right] - f(1)\left[g'(0)\right]}{g(0)^2}$$

$$= \frac{(1)\left[(3)(-1)\right] - (4)\left[-1\right]}{1^2} = 1 \qquad \boxed{E}$$

(12) $\dfrac{d^2}{dx^2}(x \sin x) =$

A. $2\cos x - x\sin x$ B. $x\cos x + \sin x$ C. $x\cos x - \sin x$

D. $x\sin x + \cos x$ E. $x\cos x - 2\sin x$

$y = x \sin x \Rightarrow y' = x \cos x + \sin x$

$\Rightarrow y'' = x(-\sin x) + \cos x + \cos x$

$= -x \sin x + 2 \cos x$

$\boxed{A}$

(13) Compute $\lim_{x\to 0} \dfrac{\sin x\ \cos x - \sin x + x\sin x}{x^2}$

A. $\frac{1}{2}$ B. DNE C. 0 D. 2 E. 1

$$\lim_{x\to 0} \frac{(\sin x)[\cos x - 1] + x\sin x}{x^2}$$

$$= \lim_{x\to 0} \frac{(\sin x)[\cos x - 1]}{x^2} + \frac{x\sin x}{x^2}$$

$$= \lim_{x\to 0} \left(\frac{\sin x}{x}\right)\left(\frac{\cos x - 1}{x}\right) + \lim_{x\to 0} \frac{\sin x}{x}$$

$$= \lim_{x\to 0} \left(\frac{\sin x}{x}\right) \lim_{x\to 0} \left(\frac{\cos x - 1}{x}\right) + \lim_{x\to 0} \frac{\sin x}{x}$$

$$= (1)(0) + 1 = 1$$

$$\boxed{E}$$

(14) $\dfrac{d}{dx}\Big(\sin^3(x) - \cos(x^3)\Big) =$

A. $3\sin^2(x)\cos(x) - 3x^2\sin(x^3)$

B. $3\sin^2(x)\cos(x) + 3x^2\sin(x^3)\cos(x^3)$

C. $3\sin^2(x)\cos(x) - 3x^2\sin(x^3)\cos(x^3)$

D. $3\sin^2(x)\cos(x) + 3x^2\cos(x^3)$

E. $3\sin^2(x)\cos(x) + 3x^2\sin(x^3)$

$$\left(\sin^3 x - \cos(x^3)\right)'$$

$$= 3\sin^2 x\cos x + (\sin(x^3))(3x^2)$$

$$= 3\sin^2 x\cos x + 3x^2\sin(x^3)$$

$$\boxed{E}$$

(15) Use Implicit Differentiation to find $\boldsymbol{y'}$ if $\boldsymbol{x^2 + 5xy - 6y^4 = 18}$

A. $\dfrac{2x+5y}{24y^3-5x}$ B. $\dfrac{2x+5y}{24y^3}$ C. $\dfrac{2x}{24y^3-5x}$

D. $\dfrac{2x+5y-18}{24y^3}$ E. $\dfrac{2x+5y-18}{24y^3-5x}$

$y = y(x)$

$$\frac{d}{dx}\{x^2+5xy-6y^4\} = \frac{d}{dx}\{18\}$$

$$\Rightarrow 2x+5\left[x\frac{dy}{dx}+y(1)\right]-24y^3\frac{dy}{dx}=0$$

$$\Rightarrow \frac{dy}{dx}\{5x-24y^3\} = -2x-5y$$

$$\frac{dy}{dx} = \frac{-2x-5y}{5x-24y^3} = \frac{-(2x+5y)}{-(-5x+24y^3)}$$

$$= \frac{2x+5y}{24y^3-5x}$$

$\boxed{A}$

(16) Use Logarithmic Differentiation to find the derivative of $\boldsymbol{f(x) = x^{\sin x}}$

A. $(\sin x)\, x^{\sin x - 1}$ **B.** $x^{\sin x}\,(\cos x)\,(\ln x)$

C. $\dfrac{\sin x}{x} + (\cos x)\,(\ln x)$ **D.** $x^{\sin x}\left[\dfrac{\sin x}{x} + (\cos x)\,(\ln x)\right]$

E. $x \cos x + \sin x$

$$\ln f(x) = \ln\{x^{\sin x}\}$$

$$\ln f(x) = (\sin x)(\ln x)$$

$$\therefore \frac{d}{dx}\{\ln f(x)\} = \frac{d}{dx}\{(\sin x)(\ln x)\}$$

$$\frac{1}{f(x)} f'(x) = (\sin x)\left[\frac{1}{x}\right] + (\ln x)\left[\cos x\right]$$

$$\Rightarrow f'(x) = f(x)\left[\frac{\sin x}{x} + (\ln x)(\cos x)\right]$$

$$= x^{\sin x}\left[\frac{\sin x}{x} + (\ln x)(\cos x)\right]$$

$$\boxed{D}$$

(17) If $\boldsymbol{f(x) = x^{\ln x}}$, find $\boldsymbol{f'(e)}$.

A. 0 B. 1 C. 2 D. 3 E. 4

Use Log. Diff.

$$\ln f(x) = \ln\{x^{\ln x}\}$$

$$\ln f(x) = (\ln x)(\ln x) = (\ln x)^2$$

$$\frac{d}{dx}\{\ln f(x)\} = \frac{d}{dx}\{(\ln x)^2\}$$

$$\frac{1}{f(x)} f'(x) = 2(\ln x)\left(\frac{1}{x}\right)$$

$$\Rightarrow f'(x) = f(x)\left[\frac{2\ln x}{x}\right]$$

$$= x^{\ln x}\left[\frac{2\ln x}{x}\right]$$

$$f'(e) = e^{\ln e}\left[\frac{2\ln e}{e}\right] = e\left[\frac{2(1)}{e}\right] = 2$$

$$\boxed{C}$$

(18) If $\boldsymbol{y = x^2 + 2^x}$, then $\boldsymbol{\dfrac{dy}{dx}} =$

A. $\boldsymbol{2x + x2^{x-1}}$ **B.** $\boldsymbol{2x + 2^x}$ **C.** $\boldsymbol{2x + 2^x \ln 2}$

D. $\boldsymbol{(2x + 2^x)\ \ln 2}$ **E.** $\boldsymbol{2x + \dfrac{2^x}{\ln 2}}$

$y' = 2x + 2^x \ln 2$ $\boxed{C}$

(19) Find $f'(1)$ if $f(x) = \ln\left[\dfrac{(3x-1)^2}{(x+1)^4}\right]$

A. 1 **B.** -1 **C.** 2 **D.** -2 **E.** 5

$$f(x) = \ln\{(3x-1)^2\} - \ln\{(x+1)^4\}$$

$$f(x) = 2\ln(3x-1) - 4\ln(x+1)$$

$$\Rightarrow f'(x) = 2\left(\frac{1}{3x-1}\right)(3) - 4\left(\frac{1}{x+1}\right)(1)$$

$$f'(1) = 2\left(\frac{1}{2}\right)(3) - 4\left(\frac{1}{2}\right)(1) = 1$$

$\boxed{A}$

(20) If $f(x) = \tan^{-1}\left(\frac{2}{x^2}\right)$, $f'(-1) =$

A. 1 B. $\frac{4}{5}$ C. $-\frac{2}{5}$ D. $\frac{2}{5}$ E. $-\frac{4}{5}$

$$f(x) = \tan^{-1}(2x^{-2})$$

$$\Rightarrow f'(x) = \frac{1}{1+(2x^{-2})^2}(2)(-2x^{-3})$$

$$f'(x) = \frac{1}{1+\left(\frac{2}{x^2}\right)^2}\left(\frac{-4}{x^3}\right)$$

$$\therefore f'(-1) = \frac{1}{5}\left(\frac{-4}{-1}\right) = \frac{4}{5} \quad \boxed{B}$$

(21) If $y = \arcsin(x) - \sqrt{1-x^2}$, $\dfrac{dy}{dx} =$

A. $\dfrac{1}{2\sqrt{1-x^2}}$ B. $\dfrac{1+x}{\sqrt{1-x^2}}$ C. $\dfrac{2}{\sqrt{1-x^2}}$ D. $\dfrac{x^2}{\sqrt{1-x^2}}$

E. $\dfrac{1}{\sqrt{1+x}}$

$y = \sin^{-1}x - (1-x^2)^{1/2}$

$\Rightarrow y' = \dfrac{1}{\sqrt{1-x^2}} - \dfrac{1}{2}(1-x^2)^{-1/2}(-2x)$

$= \dfrac{1}{\sqrt{1-x^2}} + \dfrac{x}{\sqrt{1-x^2}} = \dfrac{1+x}{\sqrt{1-x^2}}$ $\boxed{B}$

(22) Compute $\boldsymbol{y''}$ if $\boldsymbol{x^2 + y^2 = 2y + 5}$.

Implicit Diff.

A. $\dfrac{d^2y}{dx^2} = \dfrac{1}{1-y} + \dfrac{x^2}{(1-y)^2}$ B. $\dfrac{d^2y}{dx^2} = \dfrac{1}{1-y} + \dfrac{x^2}{(1-y)^3}$

C. $\dfrac{d^2y}{dx^2} = \dfrac{1+x^2}{(1-y)^2}$ D. $\dfrac{d^2y}{dx^2} = \dfrac{1+x^2}{(1-y)^3}$

E. $\dfrac{d^2y}{dx^2} = \dfrac{x^2}{1-y} - \dfrac{1}{(1-y)^3}$

$y = y(x)$

$$\frac{d}{dx}\{x^2+y^2\} = \frac{d}{dx}\{2y+5\}$$

$$2x + 2y\frac{dy}{dx} = 2\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{-2x}{2y-2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-x}{y-1} \Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{-x}{y-1}\right)$$

$$= \frac{(y-1)(-1) - (-x)\left(\frac{dy}{dx}\right)}{(y-1)^2} = \frac{-(y-1) + x\left(\frac{-x}{y-1}\right)}{(y-1)^2}$$

$$= \frac{-(y-1)}{(y-1)^2} - \frac{x^2}{(y-1)^3} = \frac{-1}{y-1} - \frac{x^2}{(y-1)^3}$$

$$= \frac{1}{1-y} + \frac{x^2}{(1-y)^3} \quad \boxed{B}$$

(23) If $\boldsymbol{y = y(x)}$ is defined implicitly by the equation $\boldsymbol{y\, e^{y^2} = 10x}$,

then $\dfrac{\boldsymbol{dy}}{\boldsymbol{dx}} =$

A. $\boldsymbol{y + 2ye^{y^2}}$ **B.** $\boldsymbol{2y^2e^{y^2} + e^{y^2}}$ **C.** $\boldsymbol{\dfrac{y}{x(2y^2+1)}}$ **D.** $\boldsymbol{\dfrac{y}{(2y^2+1)}}$

E. $\boldsymbol{\dfrac{y}{10(2y^2+1)}}$

Log. Diff Method: $\ln\left(ye^{y^2}\right) = \ln(10x)$

$\Rightarrow \ln y + \ln\left(e^{y^2}\right) = \ln(10x)$

$\Rightarrow \ln y + y^2 = \ln(10x)$ now diff. w.r.t. x

$\Rightarrow \dfrac{1}{y}\dfrac{dy}{dx} + 2y\dfrac{dy}{dx} = \dfrac{1}{x} \Rightarrow \dfrac{dy}{dx} = \dfrac{\frac{1}{x}}{\frac{1}{y}+2y}$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{\frac{1}{x}}{\frac{1+2y^2}{y}} = \dfrac{y}{x(1+2y^2)}$ $\boxed{C}$

Note: Could just do by Implicit Diff. also.

$\boxed{24}$ If $f(x) = x^3 + x + 1$, compute $(f^{-1})'(3)$

<u>Soln</u>: If $x_0 = 1 \Rightarrow y_0 = f(x_0) = f(1) = 3$

$\therefore (f^{-1})'(y_0) = \dfrac{1}{f'(x_0)}$; $f'(x) = 3x^2 + 1$

so $(f^{-1})'(3) = \dfrac{1}{f'(1)} = \dfrac{1}{3(1)^2+1} = \dfrac{1}{4}$

$\boxed{D}$

Answers

1. B **2.** A **3.** A **4.** E **5.** C **6.** E **7.** A **8.** D **9.** E
10. E **11.** E **12.** A **13.** E **14.** E **15.** A **16.** D **17.** C
18. C **19.** A **20.** B **21.** B **22.** B **23.** C **24.** D