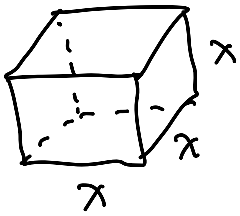
$\boxed{\text{Solutions}}$

$\boxed{1}$



Given Rate: $\dfrac{dx}{dt} = 3$ cm/sec

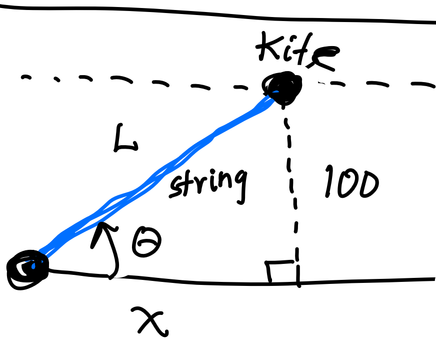
Desired Rate: $\dfrac{dV}{dt}$ when $x = 10$ cm

Equation: $V = x^3$

$$\Rightarrow \frac{dV}{dt} = 3x^2 \frac{dx}{dt} \Rightarrow \left.\frac{dV}{dt}\right|_{x=10} = 3(10)^2(3) = \underline{900 \text{ cm}^3/\text{sec}}$$

$\boxed{C}$

$\boxed{2}$



Given rate: $\dfrac{d\theta}{dt} = -\dfrac{1}{40}$ rad/s

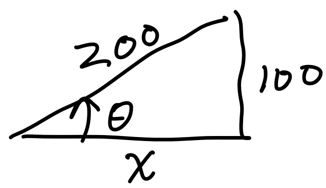
Desired rate: $\dfrac{dx}{dt}$ when $L = 200$

Equation: $\tan\theta = \dfrac{100}{x}$ ⊛

$$\Rightarrow \frac{d(\tan\theta)}{dt} = \frac{d}{dt}\left(\frac{100}{x}\right) \Rightarrow (\sec^2\theta)\frac{d\theta}{dt} = -\frac{100}{x^2}\frac{dx}{dt}$$

$$\Rightarrow \frac{dx}{dt} = -\frac{x^2\sec^2\theta}{100}\frac{d\theta}{dt}$$

Now when $L = 200$, picture is



$$\Rightarrow \cos\theta = \frac{x}{200} \Rightarrow \sec\theta = \frac{200}{x}$$

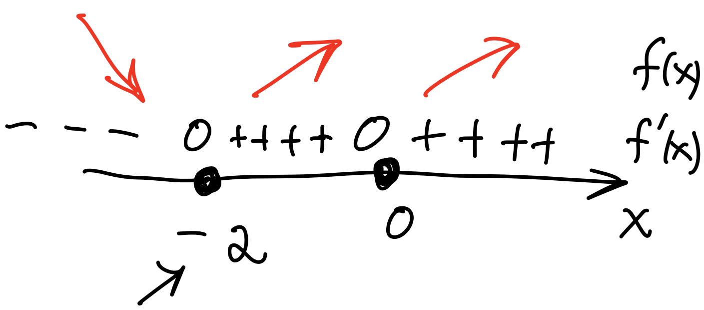
(cont'd)

$$\therefore \left.\frac{dx}{dt}\right|_{h=200} = \frac{-x^2\left(\frac{200}{x}\right)^2}{100}\left(-\frac{1}{40}\right) = \underline{10 \text{ ft/sec}}$$

$\boxed{B}$

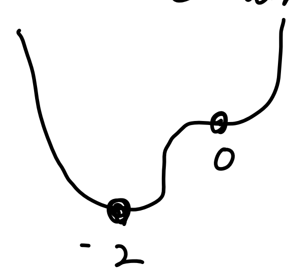
$\boxed{3}$ $f(x) = \frac{3}{8}x^4 + x^3$

$\Rightarrow f'(x) = x^2\left(\frac{3x}{2} + 3\right)$ So $f'(x) = 0$ when $x = 0$ and $x = -2$



$f'(x) = x^2\left(\frac{3x}{2} + 3\right)$

a local min occurs here, but it is also the absolute minimum occurs here since graph of $f(x)$ is



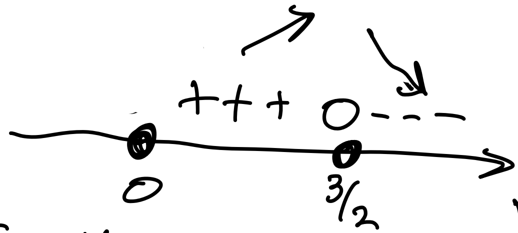
$\boxed{C}$

$\boxed{4}$

Maximize: $A = \frac{1}{2} r^2 \theta$.

Constraint: $r + r + r\theta = 6$ (*)

$\Rightarrow \theta = \frac{6-2r}{r}$ $\therefore$ Maximize $A(r) = \frac{1}{2} r^2 \left(\frac{6-2r}{r}\right)$ ✓



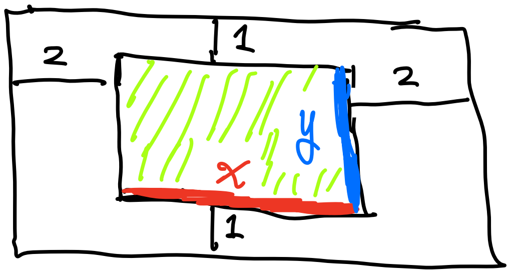
$A(r)$

$A'(r) = 3 - 2r$

Exactly one local extremum $\Rightarrow$ it is an absolute extremum. Hence abs. max occurs at $r = 3/2$ $\therefore$ Max area is $A(3/2) = \frac{9}{4}$

E

$\boxed{5}$



Minimize: $A = (y+2)(x+4)$

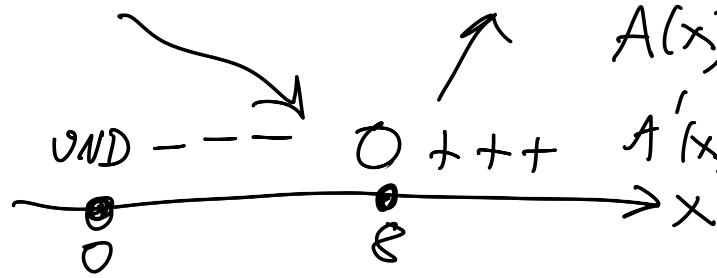
Constraint: $xy = 32$

$\Rightarrow y = \frac{32}{x}$ $\therefore$ Minimize $A(x) = \left(\frac{32}{x} + 2\right)(x+4)$

$\Rightarrow A'(x) = \frac{2(x^2 - 64)}{x^2}$ (after some algebra)

(cont'd)

5



$$A'(x) = \frac{2(x^2-64)}{x^2}$$

Exactly one local extremum i.e. it is the place where abs. min occurs.

Hence min area is $A(8) = 72$

Since $x = 8$ and $y = \frac{32}{8} = 4$

$\Rightarrow$ dimension of whole poster is

$x+4 = 8+4 = 12''$ long

$y+2 = 4+2 = 6''$ wide

A

$\boxed{6}$ Mean Value Thm (MVT):

⑥

$$\frac{f(b)-f(a)}{b-a} = f'(c) \text{ for some } c \text{ where } a<c<b$$

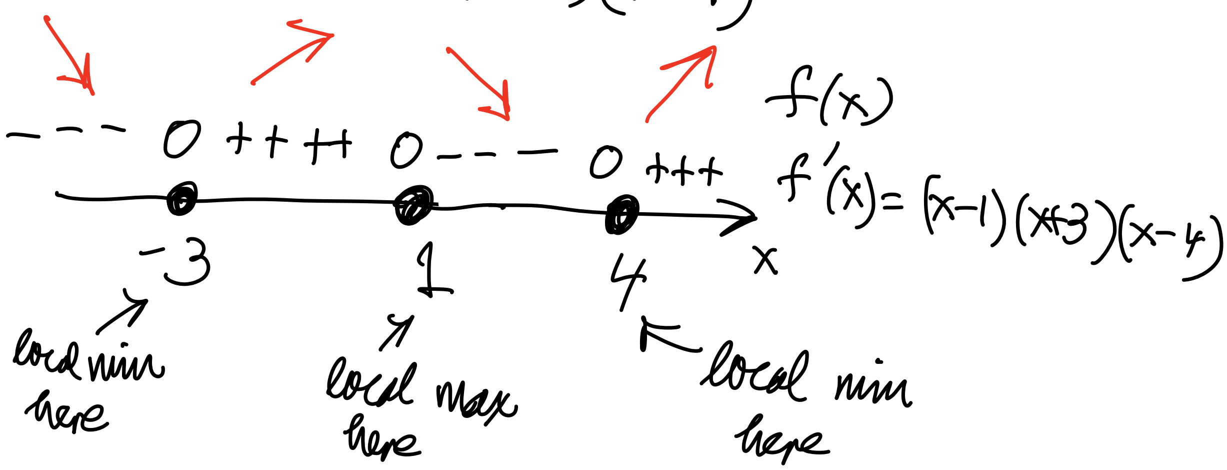
given $f(2)=1$, $f'(x)\geq 3$, Interval is $[2, 4]$ ($2 = a$, $4 = b$)

$$\Rightarrow \frac{f(4)-f(2)}{4-2} = f'(c) \geq 3 \quad \text{since}$$

$$\Rightarrow \frac{f(4)-1}{2} \geq 3 \Rightarrow f(4)\geq 7$$

$\boxed{E}$

$\boxed{7}$ $f'(x) = (x-1)(x+3)(x-4)$

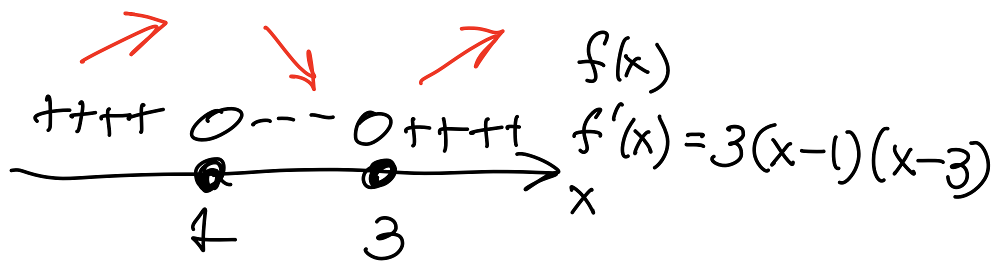


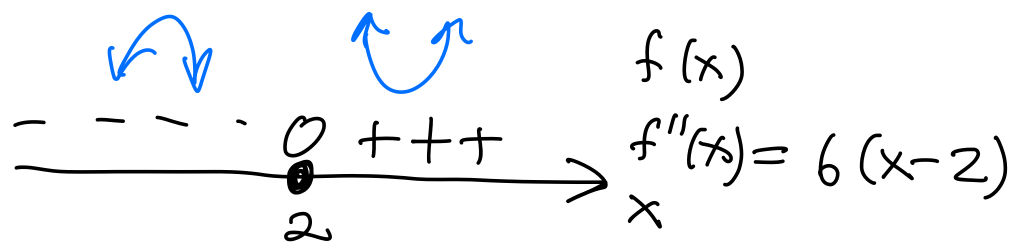
$\boxed{E}$

$\boxed{8}$ $f(x) = x^3 - 6x^2 + 9x + 30$

$\Rightarrow f'(x) = 3(x-1)(x-3)$

$f''(x) = 6(x-2)$





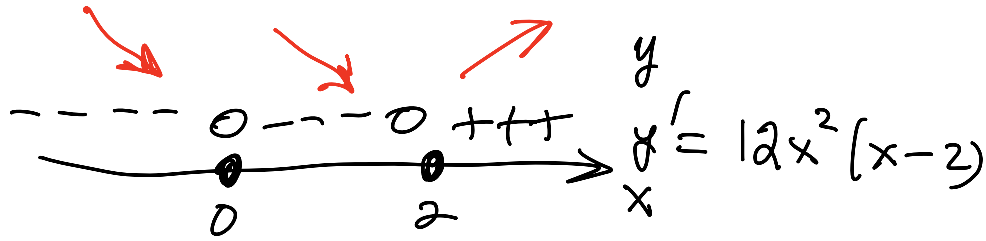
$\therefore$ On $(2,3) \Rightarrow f \searrow$ and $\cup$

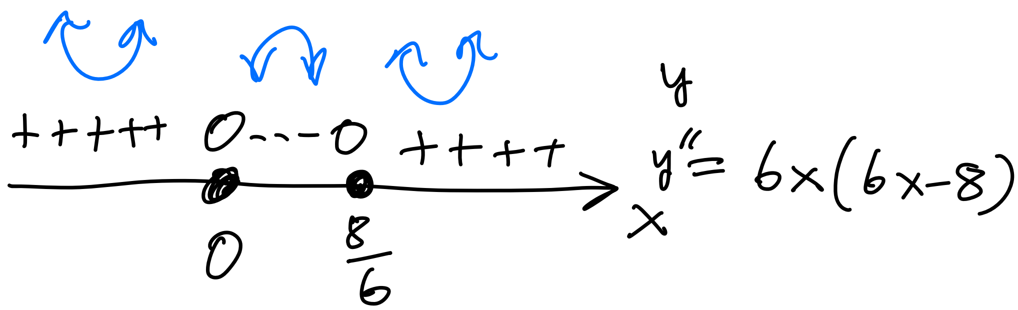
$\boxed{D}$

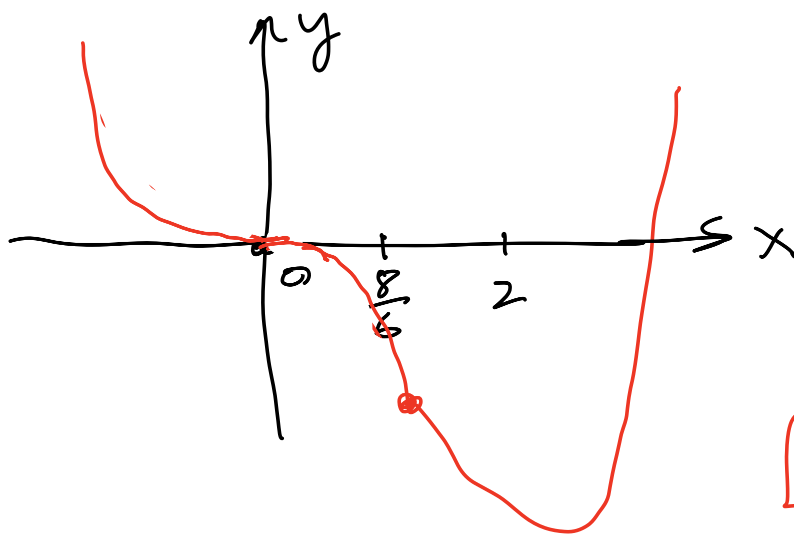
$\boxed{9}$ $y = 3x^4 - 8x^3$

$\Rightarrow y' = 12x^2(x-2)$

$y'' = 6x(6x-8)$

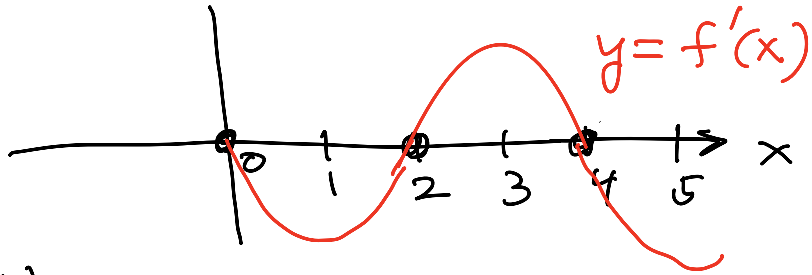






$\boxed{A}$

$\boxed{10}$



(interior) critical pts at $x=2$, $x=4$

Since f' is ↓ (decreasing) on $(0,1)\cup(3,5) \Rightarrow f''(x)<0$ there

f' is ↗ (increasing) on $(1,3) \Rightarrow f''(x)>0$ there

Thus,

↘ ↗ ↘ $f(x)$

0 − − − 0 + + + + 0 − − − $f'(x)$ (look at picture)

0 2 4 x

local min here

local max here

↶↷ ◡ ↶↷ $f(x)$

− − − − − 0 + + + + 0 − − − − − $f''(x)$

0 1 3 x

both are inflection pts

$\therefore$ $\boxed{\text{II}}$ and $\boxed{\text{III}}$ are TRUE ✓

($\boxed{\text{I}}$ is False)

$\boxed{C}$

$\boxed{11}$ $f(x) = x^2 + \frac{b}{x^3}$ (10)

$\Rightarrow f'(x) = 2x - \frac{3b}{x^4}$

$\Rightarrow f''(x) = \frac{2x^5 + 12b}{x^5}$

f has inflection pt at $x=1$ if $f(1)$ exists and $f''(1)=0$ Hence $f''(1) = \frac{2(1)^5 + 12b}{1^5} = 0$

$\Rightarrow \underline{\underline{b = -\frac{1}{6}}}$ $\boxed{B}$

$\boxed{12}$ $\lim_{x\to\infty}\left(1+\frac{3}{x}\right)^{4x} = \lim_{x\to\infty}\left[e^{\ln\left\{\left(1+\frac{3}{x}\right)^{4x}\right\}}\right]$

$= \lim_{x\to\infty}\left[e^{4x\ln\left(1+\frac{3}{x}\right)}\right] = e^{\left[\lim_{x\to\infty} 4x\ln\left(1+\frac{3}{x}\right)\right]}$

$= e^{\left[\lim_{x\to\infty}\frac{4\ln\left(1+\frac{3}{x}\right)}{\frac{1}{x}}\right]} = e^{\left[\lim_{x\to\infty}\frac{\frac{4}{\left(1+\frac{3}{x}\right)}\left(-\frac{3}{x^2}\right)}{-\frac{1}{x^2}}\right]}$

L'Hôpital's Rule

$= e^{\left[\lim_{x\to\infty}\frac{12}{\left(1+\frac{3}{x}\right)}\right]} = \underline{\underline{e^{12}}}$ $\boxed{C}$

$\boxed{13}$ $\lim\limits_{x \to 0^+} x^2 \ln(3x) = \lim\limits_{x \to 0^+} \dfrac{\ln(3x)}{\frac{1}{x^2}}$ (11)

$$\underset{\substack{\uparrow \\ \text{L'Hôpital's Rule}}}{=} \lim_{x \to 0^+} \frac{\frac{1}{x}}{-\frac{2}{x^3}} = \lim_{x \to 0^+} \frac{-x^2}{2} = 0 \quad \therefore \boxed{\text{I}} \text{ TRUE}$$

Next, $\lim\limits_{x \to \infty} \left(\frac{1}{x} - \frac{1}{\sqrt{x}}\right) = \lim\limits_{x \to \infty} \frac{1}{x} - \lim\limits_{x \to \infty} \frac{1}{\sqrt{x}}$

$$= 0 - 0 = 0$$

$\therefore$ $\boxed{\text{II}}$ TRUE

Finally, $\lim\limits_{x \to \infty} \dfrac{\sqrt{x^2+2x} + x}{x} \overset{\text{L'Hopital's Rule}}{=} \lim\limits_{x \to \infty} \dfrac{(x^2+2x)^{-1/2}(x+1) + 1}{1}$

$$= \lim_{x \to \infty} \frac{x+1}{\sqrt{x^2+2x}} + 1 = \underbrace{\lim_{x \to \infty} \frac{(x+1) + \sqrt{x^2+2x}}{\sqrt{x^2+2x}}}$$

This limit is worse. We can continue applying L'Hôpital's Rule and it keeps getting worse. Now what?

(cont'd)

Simply use techniques we learned before: (12)

$$\lim_{x\to\infty} \frac{\sqrt{x^2+2x}+x}{x} = \lim_{x\to\infty} \frac{\dfrac{\sqrt{x^2+2x}+x}{x}}{\dfrac{x}{x}}$$

$$= \lim_{x\to\infty}\left(\frac{\sqrt{x^2+2x}}{x}+1\right) = \lim_{x\to\infty}\left(\frac{\sqrt{x^2+2x}}{\sqrt{x^2}}+1\right)$$

$$= \lim_{x\to\infty}\left(\sqrt{\frac{x^2+2x}{x^2}}+1\right) = \lim_{x\to\infty}\left(\sqrt{1+\frac{2}{x}}+1\right)$$

$$= 2 \quad \therefore \boxed{\text{III}} \text{ TRUE}$$

$\boxed{\text{E}}$

13

$\boxed{14}$ Let $f(x)=e^{-x}$; $a=0$

we want to approximate $f(0.01)$.

$f'(x)=-e^{-x}$

Linear approximation of $f(x)$ at $a=0$ is

$$L(x)=f(0)+f'(0)(x-0)$$

$\therefore$ $L(x)=1+(-1)x = 1-x$

$\therefore$ $f(0.01)\approx L(0.01)=1-0.01=\underline{0.99}$

$\boxed{D}$

$\boxed{15}$ $f(x)=\dfrac{(x-1)^2}{x}=\dfrac{x^2-2x+1}{x}=x-2+\dfrac{1}{x}$

All of the Antiderivatives are $F(x)=\int\left(x-2+\frac{1}{x}\right)dx$

$\Rightarrow$ $F(x)=\dfrac{x^2}{2}-2x+\ln|x|+C$ $\leftarrow$ arbitrary constant

$\therefore$ let $C=0$ $\Rightarrow$

$\boxed{C}$

$\boxed{16}$ $v(t) = \frac{1}{t^2+1} \Rightarrow s(t) = \int \frac{1}{t^2+1}\,dt = \tan^{-1}t + C$

$\therefore$ Distance it travels from $t=1$ to $t=\sqrt{3}$ is

$$s(\sqrt{3}) - s(1) = [\tan^{-1}\sqrt{3} + C] - [\tan^{-1}1 + C]$$
$$= \tan^{-1}\sqrt{3} - \tan^{-1}1$$
$$= \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12} \checkmark$$

$\boxed{E}$

$\boxed{17}$ $f(x) = 2x + 4x^{-\frac{1}{2}} + 1$

$\therefore$ all antiderivatives are $F(x) = \int (2x + 4x^{-\frac{1}{2}} + 1)\,dx$

$$\Rightarrow F(x) = x^2 + 4\left(\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\right) + x + C = x^2 + 8x^{1/2} + x + C$$

Now $6 = F(1) = 1 + 8 + 1 + C = 10 + C \Rightarrow C = -4$

$\therefore F(x) = x^2 + 8\sqrt{x} + x - 4 \checkmark$

So $F(4) = 16 + 8(2) + 4 - 4 = \underline{\underline{32}}$

$\boxed{E}$

(15)

$\boxed{18}$ $g''(x) = 6x \Rightarrow g'(x) = 3x^2 + C_1$

$\Rightarrow g(x) = x^3 + C_1 x + C_2$ ✓

$\therefore -4 = g(0) = 0^3 + C_1 0 + C_2 \Rightarrow C_2 = -4$

$1 = g'(0) = 3(0^2) + C_1 \Rightarrow C_1 = 1$

A

$\therefore \underline{g(x) = x^3 + x - 4}$ so $g(1) = 1^3 + 1 - 4 = \underline{\underline{-2}}$

$\boxed{19}$

(a) $\int \left(4x^{-1/2} + x^{3/2}\right) dx = 4\left(\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\right) + \left(\frac{x^{3/2+1}}{\frac{3}{2}+1}\right) + C$

$= 8x^{\frac{1}{2}} + \frac{2}{5}x^{\frac{5}{2}} + C$ ✓

(b) $\int (x^2 - 3)^2 dx = \int (x^4 - 6x^2 + 9) dx$

$= \frac{x^5}{5} - 2x^3 + 9x + C$ ✓

(c) $\int \frac{6u - 1}{2u} du = \int \left(3 - \frac{1}{2u}\right) du = \int 3\, du - \frac{1}{2}\int \frac{1}{u} du$

$= 3u - \frac{1}{2}\ln|u| + C$ ✓

(cont'd)

19 (cont'd)

(d) $\int 1 + 2\sin x - 3\cos x \, dx$

$= \int dx + 2\int \sin x \, dx - 3\int \cos x \, dx$

$= (x + C_1) + 2(-\cos x + C_2) - 3(\sin x + C_3)$

$= x - 2\cos x - 3\sin x + [C_1 + 2C_2 - 3C_3]$

$= x - 2\cos x - 3\sin x + C$ ✓

(e) Recall $\frac{d}{d\theta}(\tan\theta) = \sec^2\theta$

$\therefore \int \left(\frac{\theta^2}{2} - \sec^2\theta\right) d\theta = \frac{\theta^3}{6} - \tan\theta + C$ ✓

17

$\boxed{20}$ $F'(x) = \dfrac{1}{x^2+3x} \Rightarrow \displaystyle\int \frac{1}{x^2+3x}\,dx = F(x) + k$

$\therefore \displaystyle\int \left(\frac{100}{x^2+3x} - 1\right) dx = 100\int \frac{1}{x^2+3x}\,dx - \int 1\,dx$

$$= 100\left[F(x) + k\right] - x$$

$$= 100\,F(x) + \underbrace{100\,k}_{\text{a constant, call it } c} - x$$

$$= 100\,F(x) - x + C$$

$\boxed{21}$ (a) $y = \sin^{-1}(3x) + (\sin 3x)^{-1}$

$y = \sin^{-1}(3x) + \csc(3x)$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{1}{\sqrt{1-(3x)^2}}(3) - \left(\csc(3x)\cot(3x)\right)3$

$\dfrac{dy}{dx} = \dfrac{3}{\sqrt{1-9x^2}} - 3\csc 3x \cot 3x$

$\therefore dy = \left[\dfrac{3}{\sqrt{1-9x^2}} - 3\csc 3x \cot 3x\right] dx$ ✓

(cont'd)

$\boxed{21}$ (b) $W = p2^p + 4$ (18)

$$\Rightarrow \frac{dw}{dp} = p\{2^p \ln 2\} + 2^p\{1\}$$

$$\frac{dw}{dp} = 2^p(p \ln 2 + 1)$$

$$\therefore \; dw = 2^p(p \ln 2 + 1)$$

$\boxed{22}$ (19)

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	-3	2	2	5
2	4	0	-1	π

(a) $f(x) \approx f(a) + f'(a)(x-a)$, for x near a

$\therefore$ $f(1.8) \approx f(2) + f'(2)(1.8-2)$ (let $a=2$)

$= 4 + (-1)(-0.2) = 4.2$ ✓

(b) $h(x) \approx h(a) + h'(a)(x-a)$, for x near a

since $h(x) = f(g(x))$

$\Rightarrow$ $h(x) = f(g(x)) \approx f(g(a)) + f'(g(a))g'(a)(x-a)$

(let $a=1$)

$\therefore$ $h(1.1) = f(g(1.1)) \approx f(g(1)) + f'(g(1))g'(1)(1.1-1)$

$= f(2) + f'(2)g'(1)(0.1)$

$= 4 + (-1)(5)(0.1) = 3.5$ ✓