$\boxed{\#1}$ Since $|x-3| = \begin{cases} (x-3), & \text{if } (x-3) \geq 0 \text{ i.e. } x \geq 3 \\ -(x-3), & \text{if } (x-3) < 0 \text{ i.e. } x < 3 \end{cases}$

$$\therefore \int_1^4 |x-3|\,dx = \int_1^3 |x-3|\,dx + \int_3^4 |x-3|\,dx$$

$$= \int_1^3 -(x-3)\,dx + \int_3^4 (x-3)\,dx$$

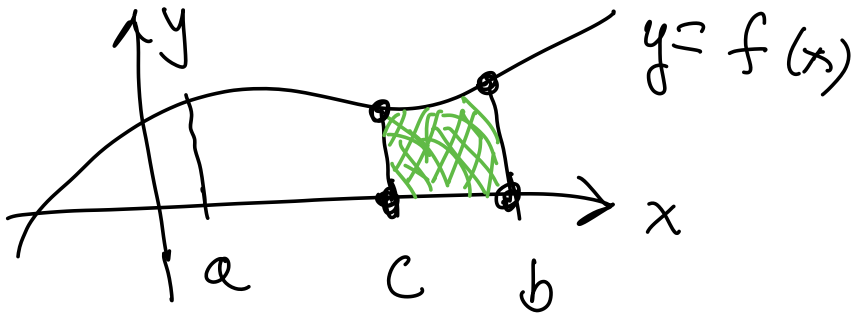
$$= \int_1^3 (-x+3)\,dx + \int_3^4 (x-3)\,dx$$

$$= \left(-\frac{x^2}{2}+3x\right)\Bigg|_{x=1}^{3} + \left(\frac{x^2}{2}-3x\right)\Bigg|_{x=3}^{4}$$

$$= \left\{\left(-\frac{9}{2}+9\right)-\left(-\frac{1}{2}+3\right)\right\} + \left\{\left(\frac{16}{2}-12\right)-\left(\frac{9}{2}-9\right)\right\}$$

$$= \frac{5}{2} \qquad \boxed{B}$$

$\boxed{\#2}$



$$\therefore \int_c^b f(x)\,dx = \int_a^b f(x)\,dx - \int_a^c f(x)\,dx$$

$\boxed{D}$

$\boxed{\#3}$ $\int_0^2 x^2 e^{x^3}\,dx = \int_0^2 x^2 e^{x^3}\,dx$

let $\boxed{u = x^3}$

then $\frac{du}{dx} = 3x^2$

so $\boxed{du = 3x^2 dx}$

$= \int_0^8 e^u \left(\frac{du}{3}\right)$

$= \frac{1}{3} e^u \Big|_{u=0}^{8}$

$= \frac{1}{3}(e^8) - \frac{1}{3}(e^0)$

$= \frac{1}{3}(e^8 - 1)$ $\boxed{A}$

$\boxed{\#4}$ $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cot x \, dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\cos x}{\sin x} \, dx$

let $\boxed{u = \sin x}$

$\frac{du}{dx} = \cos x$

$\therefore \; du = (\cos x) \, dx$

$$= \int_{\frac{1}{2}}^{1} \frac{1}{u} \, du$$

$$= \ln|u| \Big|_{u=\frac{1}{2}}^{1}$$

$$= \ln|1| - \ln\left|\frac{1}{2}\right|$$

$$= 0 - \ln\left(\frac{1}{2}\right)$$

$$= -\ln\left(\frac{1}{2}\right) \quad \boxed{B}$$

<u>Note</u>: $-\ln\left(\frac{1}{2}\right) = \ln 2$

$\boxed{\#5}$ $y(x) = \int_3^{\tan x} \sqrt{\sqrt{t} + 6t}\, dt$

FTC (Part I) i.e. Leibnitz' Rule and Chain Rule give:

$$y'(x) = \left(\sqrt{\sqrt{\tan x} + 6\tan x}\right)\left(\sec^2 x\right)$$

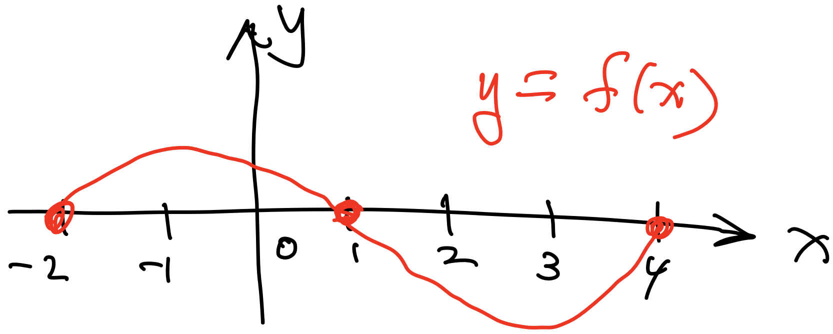
$$\therefore\ y'\left(\tfrac{\pi}{4}\right) = \left(\sqrt{\sqrt{\tan\tfrac{\pi}{4}} + 6\tan\tfrac{\pi}{4}}\right)\left(\sec^2\tfrac{\pi}{4}\right)$$

$$= \left(\sqrt{1+6(1)}\right)\left(\frac{2}{\sqrt{2}}\right)^2$$

$\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$$= \left(\sqrt{7}\right)\left(\frac{4}{2}\right) = 2\sqrt{7} \qquad \boxed{B}$$

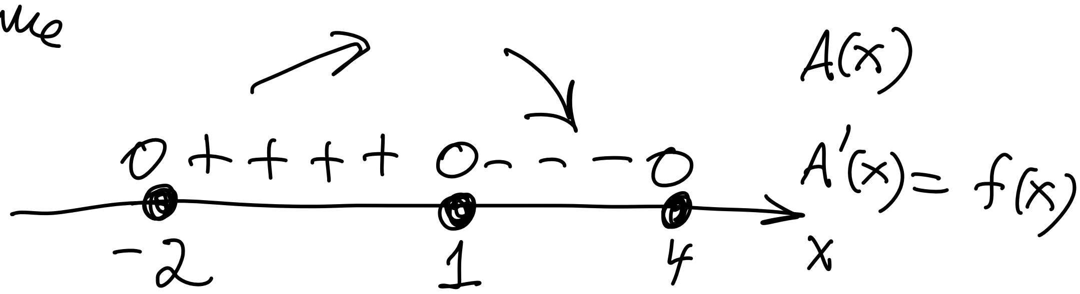
#6



$A(x) = \int_0^x f(t)\,dt$ = area under $y = f(t)$ over $0 \le t \le x$

Leibnitz' Rule $\Rightarrow A'(x) = f(x)$

Hence



A(x) has max value here

$\underline{x = 1}$ $\boxed{C}$

$\boxed{\#7}$ $(a>0)$ Given $\int_1^{\sqrt{a}} \frac{1}{x}\,dx = 3$

Thus $3 = \int_1^{\sqrt{a}} \frac{1}{x}\,dx = \ln|x| \Big|_{x=1}^{\sqrt{a}} = \ln|\sqrt{a}| - \ln|1|$

$= \ln\sqrt{a} = \frac{1}{2}\ln a$

$\therefore\ 3 = \frac{1}{2}\ln a \Rightarrow 6 = \ln a$ so $\underline{a = e^6}$

Hence $\int_1^{a} \frac{1}{x}\,dx = \int_1^{e^6} \frac{1}{x}\,dx = \ln|x| \Big|_{x=1}^{e^6}$

$= \ln|e^6| - \ln|1|$

$= 6 - 0 = 6$ $\boxed{B}$

$\boxed{\#8}$ $\boxed{y(t) = y_0 e^{kt}}$ 40% decays in 50 days.

i.e. 60% remains after 50 days; i.e. $0.60 y_0$

Hence $0.60\cancel{y_0} = y(50) = \cancel{y_0} e^{k(50)}$

$$\Rightarrow \ln(0.60) = k(50)$$

$$\Rightarrow k = \frac{\ln(0.60)}{50}$$

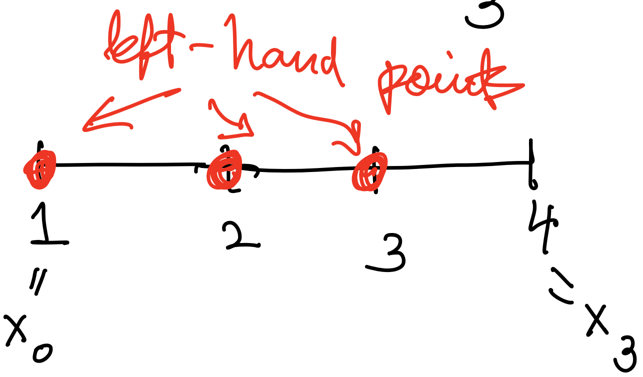
$\therefore$ Half-life is $T_{\frac{1}{2}} = \dfrac{\ln(\frac{1}{2})}{k} = \dfrac{\ln(\frac{1}{2})}{\left\{\dfrac{\ln(0.60)}{50}\right\}}$

So $T_{\frac{1}{2}} = 50 \dfrac{\ln(0.50)}{\ln(0.60)}$ $\boxed{B}$

$\boxed{\#9}$ $y = f(x) = \sqrt{x}$, $[\underset{a}{1}, \underset{b}{4}]$

$n = 3$

$$\therefore \Delta x = \frac{b-a}{n} = \frac{4-1}{3} = 1 \checkmark$$



Left Riemann Sum

$$\therefore \sum_{k=1}^{3} f(x_k^*)\Delta x = \{f(1)\cdot 1\} + \{f(2)\cdot 1\} + \{f(3)\cdot 1\}$$

$$= \underline{\sqrt{1} + \sqrt{2} + \sqrt{3}}$$

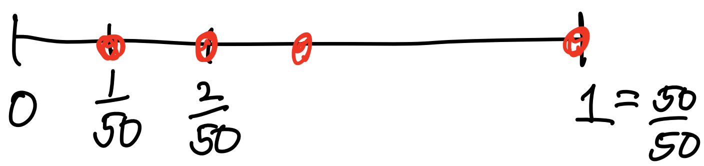
$\boxed{A}$

$\boxed{\#10}$ Given $\displaystyle\sum_{k=1}^{50} f(x_k^*)\Delta x = \frac{1}{50}\left(\sqrt{\frac{1}{50}} + \sqrt{\frac{2}{50}} + \cdots + \sqrt{\frac{50}{50}}\right)$

This $\Rightarrow$ 50 subintervals, each has length $\Delta x = \frac{1}{50}$

$\therefore$ Interval $[0, 1]$ and $f(x) = \sqrt{x}$

and Riemann sum is a Right Riemann Sum



$\therefore$ Riemann Sum approximates

$$\int_0^1 \sqrt{x}\,dx$$

$\boxed{B}$