Problem Set # 7

1. Maximize $f(x, y, z) = -2x + y + z$ subject to the constraints $x^2 + y^2 = 8$ and $y - z = 1$.

2. Consider this system of equations
$$\begin{align*}
t^2 + u^2 + x + 2z + e^{2y} &= 5 \\
y^3 + ty + x^2 + u + z &= 0
\end{align*}$$
Can this system of equations be solved for $u$ and $y$ in terms of $t, x$ and $z$ in a neighborhood of $(t_0, x_0, y_0, z_0, u_0) = (1, 0, 0, 1, -1)$?


4. If $\vec{b}(t) = x_1(t) \hat{i} + y_1(t) \hat{j}$ and $\vec{c}(t) = x_2(t) \hat{i} + y_2(t) \hat{j}$ are differentiable functions, prove that
$$\frac{d}{dt} [\vec{b}(t) \cdot \vec{c}(t)] = \vec{b}(t) \cdot \vec{c}'(t) + \vec{b}'(t) \cdot \vec{c}(t).$$

5. The International Space Station (ISS) is the largest and most complex international scientific project in history. It currently orbits the Earth at an altitude of (approximately) 353 km. Estimate the period of its orbit (in minutes).

Note: Use the fact that the moon’s period is 27.3 days, its distance from the center of the Earth is approximately 238,854 miles and the radius of the Earth is about 4000 miles.