Submitting HW Tips

HW # 7

1 **TRUE or FALSE** Question: If $S : \mathbb{R}^3 \to \mathbb{R}^2$ is a Linear Transformation, (**LT**), with

$$S\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \end{bmatrix} \text{ and } S\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}, \text{ then } \; S\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}.$$

2 **MULTIPLE CHOICE** Question: If $T : \mathcal{P}_2 \longrightarrow \mathbb{R}^2$ is the Linear Transformation (**LT**) where

$$T(4+x^2) = \begin{bmatrix} 2 \\ -3 \end{bmatrix} \quad \text{and} \quad T(6x - x^2) = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

then $T(2 - 9x + 2x^2) = ?$ (Justify your answer.)

A. $\begin{bmatrix} 1 \\ -3 \end{bmatrix}$ **B.** $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ **C.** $\begin{bmatrix} 2 \\ -2 \end{bmatrix}$ **D.** $\begin{bmatrix} -2 \\ -4 \end{bmatrix}$ **E.** Cannot be determined

3 Find $\left(\frac{1}{12} A^2\right)^{-1}$ if $A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 0 \\ 2 & 3 & 0 \end{bmatrix}$.

4 Let $\mathcal{B} = \left\{ 2,\, (x+x^2),\, (1+x)^2 \right\}$ be an ordered basis for $\mathcal{P}_2$, and $p(x) = 1 - 2x + x^2$ and $q(x) = 3x^2 - 1$. Compute the following:

(a) $\big[p(x)\big]_{\mathcal{B}} + \big[q(x)\big]_{\mathcal{B}}$ (b) $\Big|\Big|\big[p(x)\big]_{\mathcal{B}}\Big|\Big|$ (c) $\big[p(x)\big]_{\mathcal{B}} \bullet \big[q(x)\big]_{\mathcal{B}}$

5 Let $T : \mathcal{P}_2 \longrightarrow \mathbb{R}^2$ be the transformation defined by

$$T(p(x)) = \begin{bmatrix} p(1) \\ p'(1) \end{bmatrix}, \quad \text{i.e.} \quad T(a + bx + cx^2) = \begin{bmatrix} (a+b+c) \\ (b+2c) \end{bmatrix}.$$

(a) Show that T is a Linear Transformation (**LT**).

(b) Find the matrix representation M_T for T (Standard ordered basis for $\mathcal{P}_2$ and $\mathbb{R}^2$).

6 If A and Q are $n \times n$ matrices and Q is invertible, show that

(a) $\left(Q\,\mathbf{A}\,Q^{-1}\right)^2 = Q\,\mathbf{A^2}\,Q^{-1}$.

(b) If A is also invertible, then $\left(Q\,\mathbf{A}\,Q^{-1}\right)^{-1} = Q\,\mathbf{A^{-1}}\,Q^{-1}$.