Submitting HW Tips

HW #11

1 Compute the flux of $\overrightarrow{\mathbf{F}}$ across S, where $\overrightarrow{\mathbf{F}}(x, y, z) = y\mathbf{i} - x\,\mathbf{j} + z\,\mathbf{k}$ and S is that part of the paraboloid $z = 9 - x^2 - y^2$ which lies above the plane $z = 5$ and $\overrightarrow{\mathbf{N}}$ is the upward unit normal. What is the value of $\iint_S \left(\overrightarrow{\mathbf{F}} \bullet \overrightarrow{\mathbf{N}}\right)\, dS$?

2 Evaluate the line integral $I = \int_C x^2 y\, dx + y^2\, dy$, along the simple closed curve C that is the positively oriented boundary of the region between $y = 4 - x^2$ and $y = 0$.

3 Compute the line integral $J = \int_C (-5xy)\ dy + (x^3 + \cos^2 x - 4y)\, dx$, where C is the positively oriented boundary of the rectangle $R = [0,\, 2] \times [0,\, 3]$.

4 Using **Green's Theorem**, find the value of the line integral $K = \int_C y\, dx + (x^2 + y^2)\, dy,$ where C is the circle $(x-3)^2 + y^2 = 9$ traversed in a positive direction. *Hint*: Use centroids.

5 Let S be the surface $z = x^2 + y^2$ below the plane $z = 4$ with downward normal $\overrightarrow{\mathbf{n}}$ and $\overrightarrow{\mathbf{F}}(x, y, z) = (-yz,\, xz,\, y)$. Compute $\iint_S \left(\nabla \times \overrightarrow{\mathbf{F}}\right) \bullet d\overrightarrow{\mathbf{S}}$, using **Stokes' Theorem**.

6 Let $\overrightarrow{\mathbf{F}} = x\,\mathbf{i} + y\,\mathbf{j} - 3z\,\mathbf{k}$. Compute $\iint_S \overrightarrow{\mathbf{F}} \bullet d\overrightarrow{\mathbf{S}}$, where S is the closed surface consisting of that part of the cone $z = \sqrt{x^2 + y^2}$ below the plane $z = 3$, including the top, with outward normal. Compute $\iint_S \overrightarrow{\mathbf{F}} \bullet d\overrightarrow{\mathbf{S}}$ directly by using the **Divergence Theorem**.

7 **Section 8.3** (Page 459)**:** #1.

8 Let S be the oriented surface surface consisting of that part of the sphere $x^2 + y^2 + z^2 = 13$ above the plane $z = -3$, with outward unit normal $\overrightarrow{\mathbf{N}}$. If $\overrightarrow{\mathbf{F}} = \Big(2y, -2x,\, \cos(xyz)\Big)$, evaluate the surface integral $\iint_S \left(\nabla \times \overrightarrow{\mathbf{F}}\right) \bullet d\overrightarrow{\mathbf{S}}$.