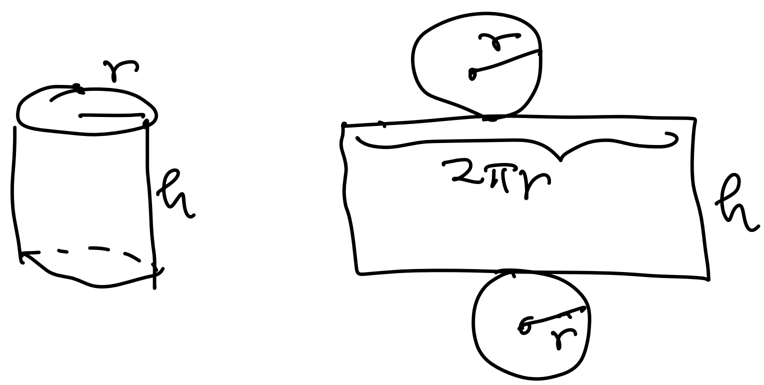
$\boxed{\text{MM-01}}$

(1.1)



<u>Minimize</u>: $A = 2(\pi r^2) + (2\pi r)h$

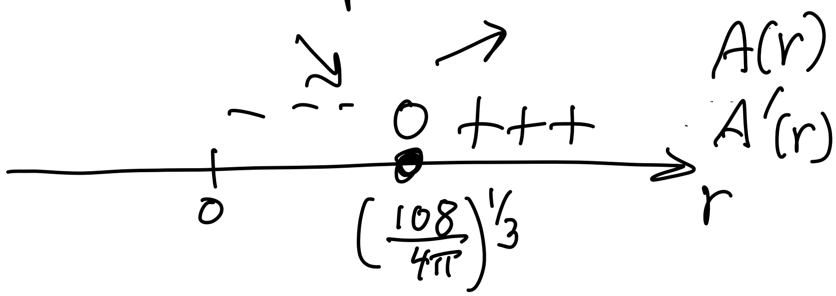
<u>Constraint</u>: volume $= \pi r^2 h = 54$

solve for h to get $h = \dfrac{54}{\pi r^2}$

Hence $A(r) = 2\pi r^2 + (2\pi r)\left(\dfrac{54}{\pi r^2}\right)$

$\therefore$ Minimize $A(r) = 2\pi r^2 + \dfrac{108}{r}$ ✓

$\Rightarrow A'(r) = \dfrac{4\pi r^3 - 108}{r^2}$



(cont'd)

I.2

$\therefore$ minimum of A occurs at

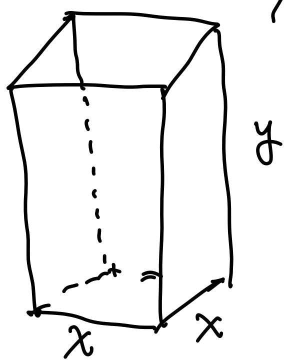
$$r = \left(\frac{108}{4\pi}\right)^{1/3} = \frac{3}{\pi^{1/3}} \checkmark$$

And since $h = \dfrac{54}{\pi r^2}$

$$\Rightarrow \quad h = \frac{54}{\pi\left(\frac{3}{\pi^{1/3}}\right)^2} = \frac{6}{\pi^{1/3}} \checkmark$$

Thus diameter of can = height of can to minimize aluminum!

MM-02



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Maximize: $V = x^2 y$

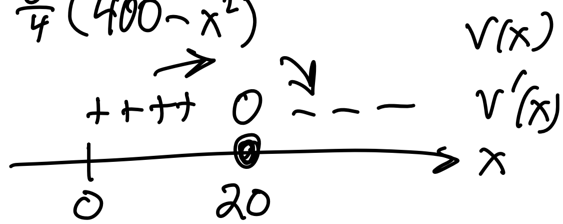
Constraint: $A = x^2 + 4xy = 1200$

Solve for y $\Rightarrow$ $y = \dfrac{1200 - x^2}{4x}$

$\therefore$ $V(x) = x^2\left[\dfrac{1200 - x^2}{4x}\right]$. Hence we want to

maximize $V(x) = \dfrac{x}{4}(1200 - x^2)$ ✓

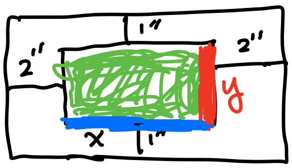
$V'(x) = \dfrac{3}{4}(400 - x^2)$



$\therefore$ max V occurs when $x = 20$

so $\boxed{V(20) = 4000 \text{ cm}^3}$

MM-03



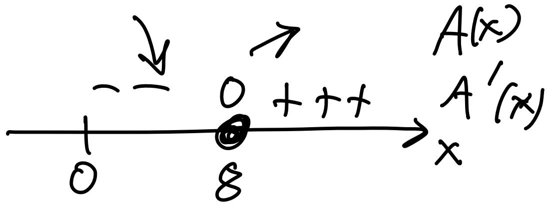
Minimize: $A = (y+2)(x+4)$

Constraint: $xy = 32$

Solve for $y \Rightarrow y = \frac{32}{x}$ so A becomes

Minimize $A(x) = \left(\frac{32}{x} + 2\right)(x+4)$ ✓

$$\Rightarrow A'(x) = \frac{2(x^2-64)}{x^2}$$



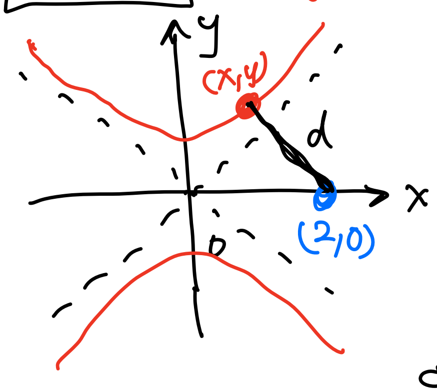
$\Rightarrow$ min A occurs when $x = 8$

so smallest possible area is

$$\boxed{A(8) = 72 \text{ in}^2}$$

$\boxed{\text{MM-04}}$

$y^2 - x^2 = 4$



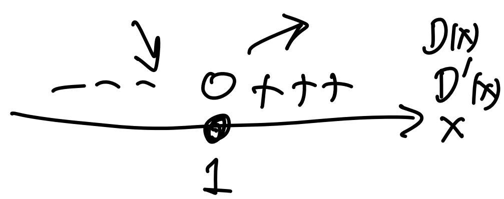
Minimize: $d = \sqrt{(x-2)^2 + (y-0)^2}$

Constraint: $y^2 - x^2 = 4$

Note that since $d \geq 0$, we can first minimize d^2 and when done, take $\sqrt{\cdot}$ of answer! (simplifies work)

so minimize $d^2 = (x-2)^2 + y^2 = (x-2)^2 + [4 + x^2]$
(using constraint)

let $D(x) = (x-2)^2 + [4+x^2] \Rightarrow D'(x) = 4(x-1)$

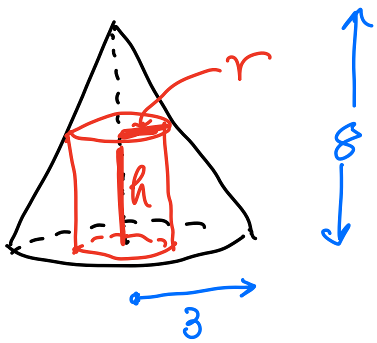


$\Rightarrow$ min D occurs when $x = 1$

$\therefore$ When $x = 1$, since $y^2 - x^2 = 4$
$\Rightarrow y = \pm\sqrt{5}$. Hence there are <u>two</u> nearest pts $(1, \pm\sqrt{5})$.

And since $D(1) = 6$, the distance is $d = \sqrt{6}$

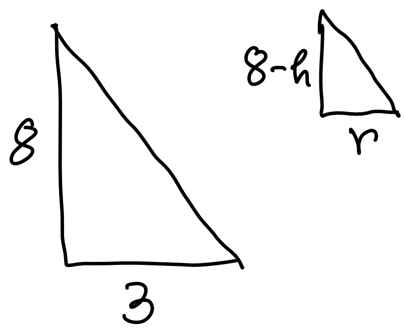
MM-05



Maximize: $V = \pi r^2 h$

Now we need to eliminate either r or h so that V is a fcn of 1 variable.

Use Similar Δ's:

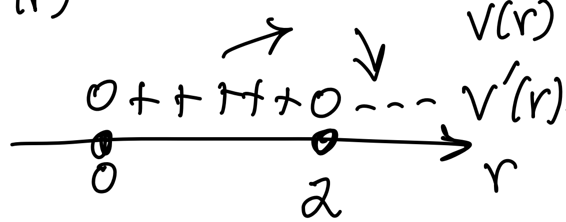


$$\Rightarrow \frac{8}{3} = \frac{8-h}{r}$$

Solve for $h \Rightarrow h = \dfrac{24-8r}{3}$

$$\therefore V(r) = \pi r^2 \left[\frac{24-8r}{3}\right] \checkmark$$

maximize $V(r)$

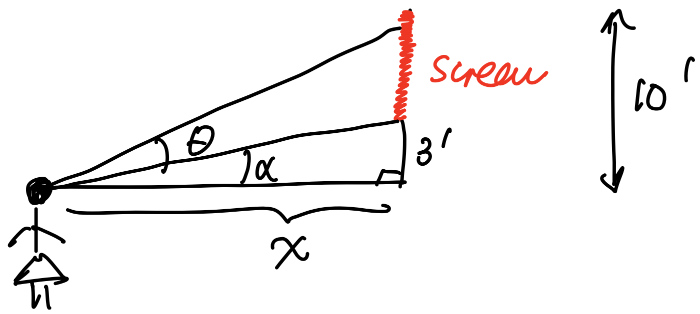


$V'(r) = 8\pi r(2-r)$

Max vol. occurs when $r = 2$

$$\therefore V(2) = \frac{32\pi}{3} \text{ ft}^3$$

MM-06



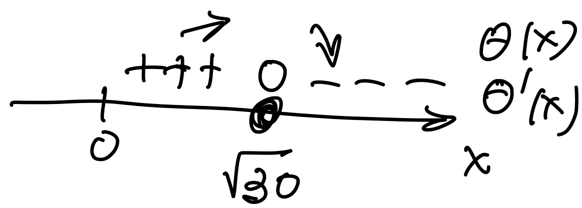
Now $\tan(\theta+\alpha) = \frac{10}{x} \Rightarrow \theta + \alpha = \tan^{-1}\left(\frac{10}{x}\right)$

Also $\tan\alpha = \frac{3}{x} \Rightarrow \alpha = \tan^{-1}\left(\frac{3}{x}\right)$

So $\theta = \tan^{-1}\left(\frac{10}{x}\right) - \alpha = \tan^{-1}\left(\frac{10}{x}\right) - \tan^{-1}\left(\frac{3}{x}\right)$

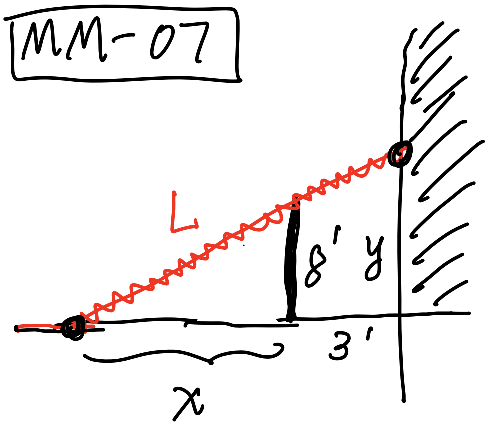
$\therefore$ Maximize: $\theta(x) = \tan^{-1}\left(\frac{10}{x}\right) - \tan^{-1}\left(\frac{3}{x}\right)$

$\Rightarrow \theta'(x) = \dfrac{7(30-x^2)}{(x^2+100)(x^2+9)}$ (after some algebra)



$\Rightarrow$ Max angle occurs when $x = \sqrt{30}$ ft

MM-07

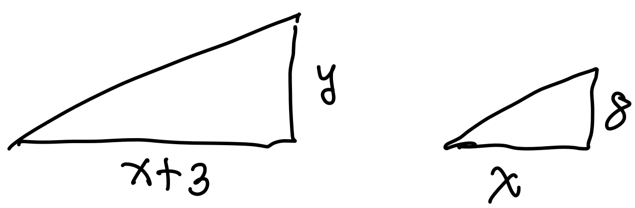


Minimize: $L = \sqrt{(x+3)^2 + y^2}$

Since $L \geq 0$, it is easier to minimize L^2, and when done take $\sqrt{\cdot}$ of answer.

so let $F = (x+3)^2 + y^2$. Need to eliminate a variable.

Use Similar Δ's:



$\Rightarrow \dfrac{y}{x+3} = \dfrac{8}{x}$

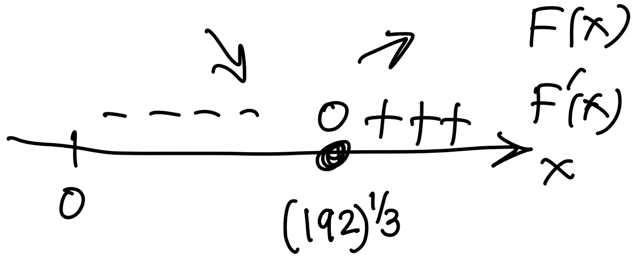
$\Rightarrow y = \dfrac{8(x+3)}{x}$

Hence minimize $F(x) = (x+3)^2 + \left[\dfrac{8(x+3)}{x}\right]^2$ ✓

$$\Rightarrow F'(x) = \frac{2(x+3)(x^3 - 192)}{x^3}$$

(cont'd)

7.2



F is minimized when $x = (192)^{1/2}$

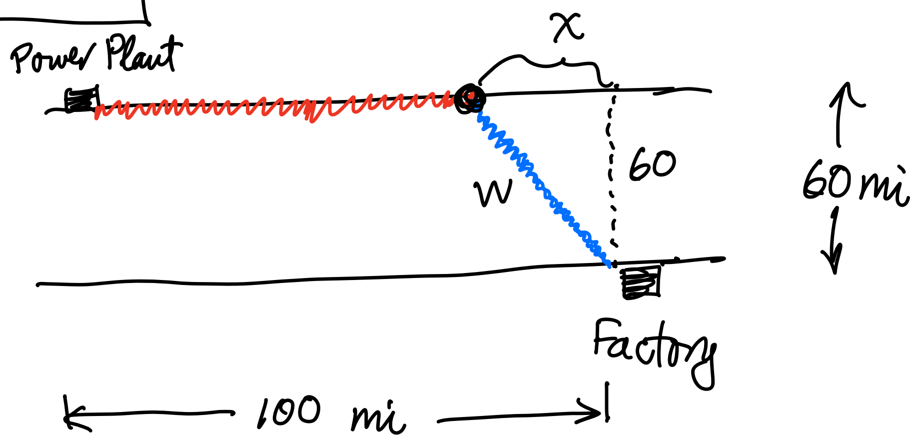
$\therefore$ since $L = \sqrt{F(x)}$

So min $L = \sqrt{F((192)^{1/3})} \approx \sqrt{224.77\ldots}$

≈ 15 ft

MM-08

8.1



Cable cost under water is $1.3w$

Cable cost along bank is $0.5(100-x)$

$\therefore$ Minimize: $C = 1.3w + 0.5(100-x)$

Need to eliminate either w or x

Note that $w^2 = x^2 + 60^2$ so $w = \sqrt{x^2+60^2}$

Hence minimize $C(x) = 1.3\sqrt{x^2+60^2} + 0.5(100-x)$

(cont'd)

8.2

$$\Rightarrow C'(x) = \frac{1.3x - 0.5\sqrt{x^2+60^2}}{\sqrt{x^2+60^2}}$$

$$C'(x) = 0 \iff 1.3x - 0.5\sqrt{x^2+60^2} = 0$$

$$\iff 1.3x = 0.5\sqrt{x^2+60^2}$$

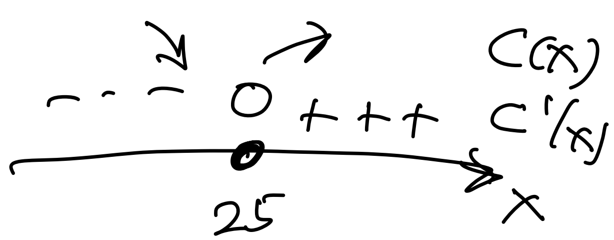
$$\iff (1.3x)^2 = \left\{0.5\sqrt{x^2+60^2}\right\}^2$$

$$\iff 1.69x^2 = 0.25(x^2+60^2)$$

$$\iff 1.44x^2 = (0.25)(60^2)$$

$$\Rightarrow x = \sqrt{\frac{(0.25)(60)^2}{1.44}}$$ (ignore negative values of x)

$$\Rightarrow x = 25$$



(cont'd)

min cost occurs at $x = 25$ (83)

$\therefore$ $C(25) = \$122$ million dollars!

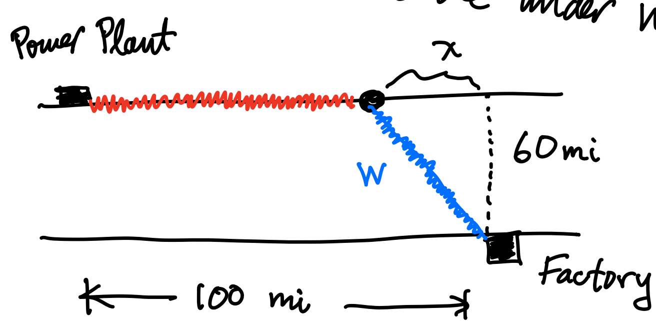
SEE NEXT PAGE
FOR INTERESTING
TWIST ON THIS PROBLEM...

Interesting Problem Related to Cable Problem

(8.4)

Suppose the cable costs were reversed:

cost to run cable along bank is \$1.3 million/mi while cost to run cable under water is \$0.5 million/mi.



Just as before, we want to

Minimize: Total Cost $C = 0.5w + 1.3(100-x)$

and since $w^2 = x^2 + 60^2$

$\Rightarrow$ minimize $C(x) = 0.5\sqrt{x^2+60^2} + 1.3(100-x)$ ✓

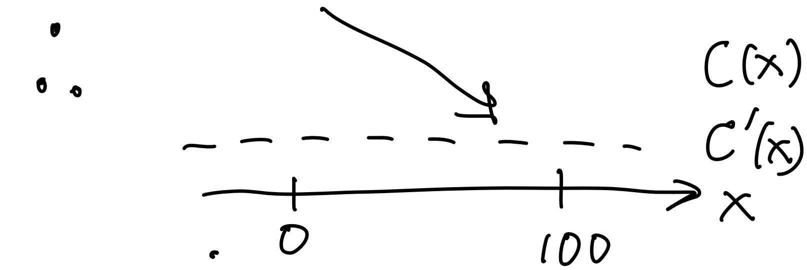
Note that $0 \le x \le 100$

Now $$C'(x) = \frac{x - 2.6\sqrt{x^2+60^2}}{2\sqrt{x^2+60^2}}$$ ✓

In this case $C'(x) \neq 0$. In fact, $C'(x) < 0$ for all x!

(cont'd)

8.5



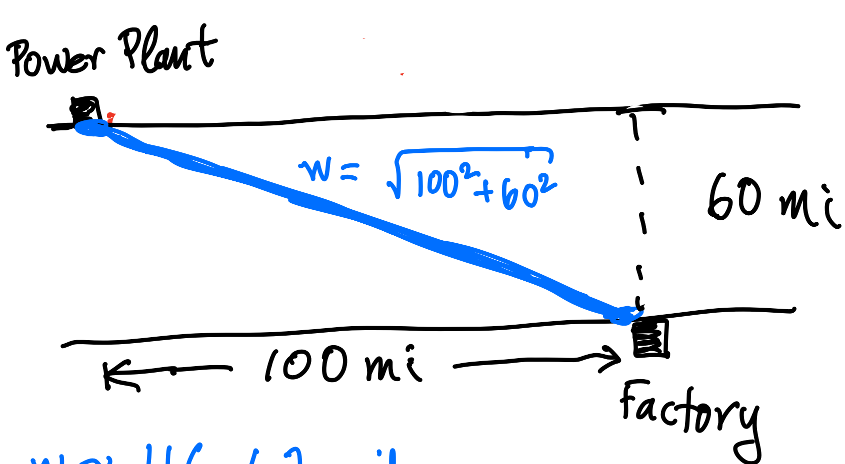
There is **no** local extremum.

$\therefore$ absolute extrema occurs at an endpt. Check endpts:

$C(0) = \$160$ million

$C(100) = 0.5\sqrt{100^2 + 60^2} \approx \$58.31$ million dollars

$\therefore$ Minimum costs occur when $x = 100$

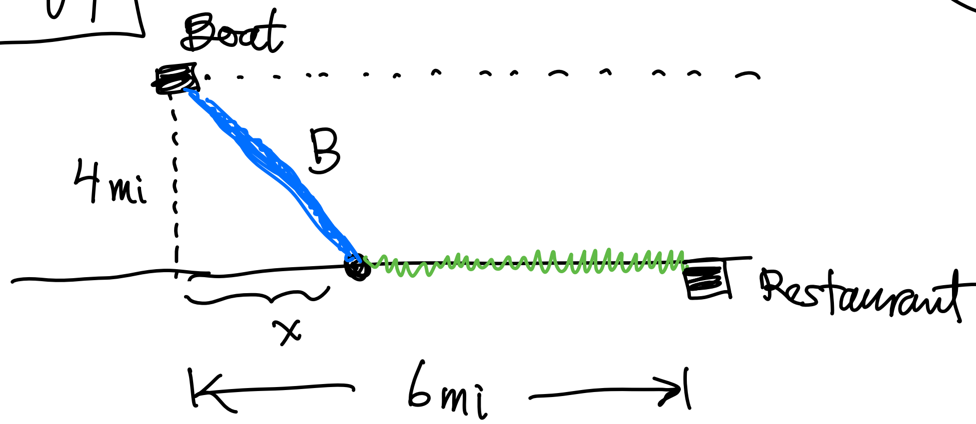


$w \approx 116.62$ miles

(cable runs under water the entire way)

MM-09

9.1



Travel time on water is $\dfrac{\text{distance}}{\text{speed}} = \dfrac{B}{2}$

Travel time on land is $\dfrac{\text{distance}}{\text{speed}} = \dfrac{6-x}{3}$

$\therefore$ <u>Minimize</u>: $T = \dfrac{B}{2} + \dfrac{6-x}{3}$

(need to eliminate a variable)

Note that $B^2 = x^2 + 4^2$, so $B = \sqrt{x^2+16}$

Hence minimize $T(x) = \dfrac{\sqrt{x^2+16}}{2} + \dfrac{6-x}{3}$ ✓

(cont'd)

(9.2)

$$\Rightarrow T'(x) = \frac{3x - 2\sqrt{x^2+16}}{6\sqrt{x^2+16}}$$

$$T'(x) = 0 \iff 3x = 2\sqrt{x^2+16}$$

$$\iff 9x^2 = 4(x^2+16)$$

$$\iff 5x^2 = 64$$

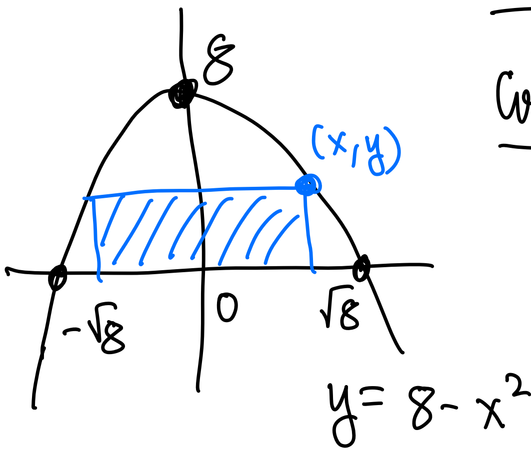
$$\Rightarrow x = \sqrt{\frac{64}{5}} = \frac{8}{\sqrt{5}}$$

ignore negative values of x

$\therefore$ Minimum time is

$$T\left(\frac{8}{\sqrt{5}}\right) = \frac{10}{3\sqrt{5}} + 2 \approx 3.5 \text{ hrs}$$

$\boxed{\text{MM-10}}$



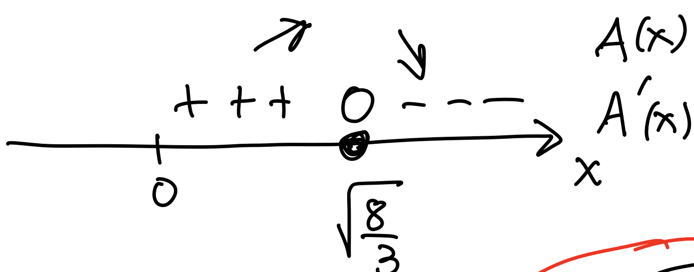
Maximize: $A = 2xy$

Constraint: $y = 8 - x^2$ $\quad x, y > 0$

$\therefore A = 2x[8 - x^2]$

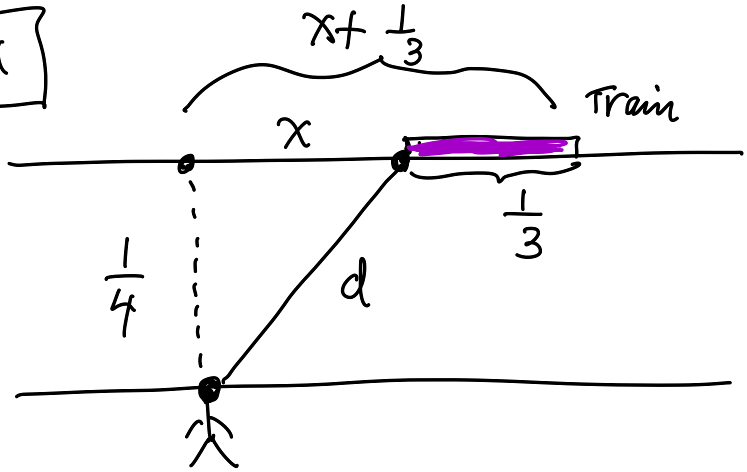
So maximize: $A(x) = 2x(8 - x^2)$ ✓

$A'(x) = 2(8 - 3x^2)$



$\Rightarrow$ max area when $x = \sqrt{\frac{8}{3}}$

$\therefore A\left(\sqrt{\frac{8}{3}}\right) = \frac{32}{3}\sqrt{\frac{8}{3}}$



Time to reach back of train is

$$t = \frac{\text{distance}}{\text{speed}} = \frac{\left(x+\frac{1}{3}\right)}{20}$$

Speed of man is $v = \frac{\text{(his) distance}}{\text{time}} = \frac{d}{t}$

Hence <u>Minimize</u>: $v = \dfrac{d}{\left\{\frac{x+\frac{1}{3}}{20}\right\}}$

Need to eliminate a variable.

Note that $d^2 = x^2 + \left(\frac{1}{4}\right)^2$,

so $d = \sqrt{x^2 + \frac{1}{16}}$

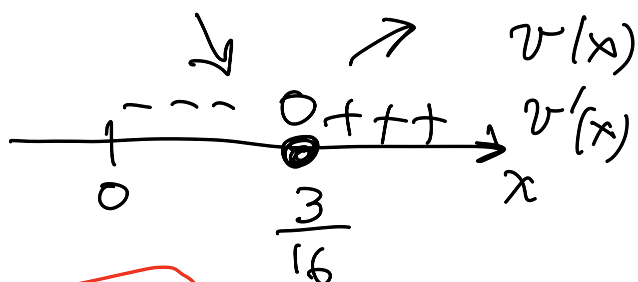
(cont'd)

11.2

Finally, we want to

<u>Minimize</u>: $v(x) = \dfrac{\sqrt{x^2 + \frac{1}{16}}}{\left\{\dfrac{\left(x + \frac{1}{3}\right)}{20}\right\}}$ ✓

$$\Rightarrow v'(x) = \frac{15(16x - 3)}{(3x+1)^2 (16x^2+1)^{1/2}}$$ (after some algebra)



Min velocity occurs when $x = \frac{3}{16}$

$$\therefore v\left(\frac{3}{16}\right) = \frac{\sqrt{\left(\frac{3}{16}\right)^2 + \frac{1}{16}}}{\left\{\dfrac{\frac{3}{16} + \frac{1}{3}}{20}\right\}} = 12 \text{ mph}$$