Study Guide # 2

<u>Chapter 14</u>

1. Double integrals; vertically and horizontally simple regions, iterated integrals; double integrals in polar coordinates ($dA = r\,dr\,d\theta$)

2. Applications of double integrals: areas between curves, volumes, surface area $S = \int\int_R \sqrt{f_x^2 + f_y^2 + 1}\; dA$.

3. Changing the order of integration in double integrals.

4. Triple integrals; iterated triple integrals; applications of triple integrals: volumes, mass $m = \int\int\int_D \delta(x,y,z)\,dV$.

5. Triple integrals in Rectangular, Cylindrical, and Spherical Coordinates:

   (a) Rectangular Coordinates: $dV = dz\,dy\,dx$ or $dV = dz\,dx\,dy$, etc

   (b) Cylindrical Coordinates: $\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = z \end{cases}$ $\quad dV = r\,dz\,dr\,d\theta$

   (c) Spherical Coordinates: $\begin{cases} x = \rho\sin\phi\cos\theta \\ y = \rho\sin\phi\sin\theta \\ z = \rho\cos\phi \end{cases}$ $\quad dV = \rho^2\sin\phi\,d\rho\,d\phi\,d\theta$

<u>Chapter 15</u>

1. Vector fields; divergence of a vector field div $\vec{\mathbf{F}}$ (i.e., $\nabla\cdot\vec{\mathbf{F}}$); curl of a vector field curl $\vec{\mathbf{F}}$ (i.e., $\nabla\times\vec{\mathbf{F}}$); Laplacian of f ($\text{div}\nabla f = \nabla^2 f = f_{xx} + f_{yy} + f_{zz}$).

2. Conservative vector fields $\vec{\mathbf{F}} = \nabla f$; how to determine if $\vec{\mathbf{F}}$ is conservative : check that curl $\vec{\mathbf{F}} = \vec{\mathbf{0}}$ (if region has no "holes").

3. Line integrals: $\int_C f(x,y,z)\,ds$, $\int_C \vec{\mathbf{F}}\cdot\vec{\mathbf{dr}}$, $\int_C M\,dx + N\,dy + P\,dz$; Fundamental Theorem of Line Integrals: $\int_C \nabla f\cdot\vec{\mathbf{dr}} = f(P_1) - f(P_0)$; independence of path (check if $\vec{\mathbf{F}} = \nabla f$ or curl $\vec{\mathbf{F}} = \vec{\mathbf{0}}$) ; applications to work $W = \int_C \vec{\mathbf{F}}\cdot\vec{\mathbf{dr}}$.

4. Green's Theorem : $\int_C M(x,y)\,dx + N(x,y)\,dy = \int\int_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dA$.